Risk Perception and Loan Underwriting in Securitized Commercial Mortgages

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We use model-implied volatility to proxy for property risk perceptions in the commercial real estate (CRE) lending market. While loan-to-value ratios (LTVs) unconditionally decreased following the Global Financial Crisis, LTVs conditioned on implied volatility and other theoretically motivated fundamental determinants of optimal leverage show no conclusive trend before or after the crisis. Taking reported property and loan attributes at face value, we find no clear pattern of unwarranted credit being extended to CRE assets. We conclude that systematically higher LTV decisions pre-crisis would have primarily stemmed from risk misperceptions rather than imprudent practices. Our findings suggest that the aggregate LTV level should be interpreted as a proxy for lending standards only after controlling for aggregate risk perceptions, among a host of asset and lending market factors. Our findings also highlight the importance of measuring and tracking aggregate risk perceptions in informing regulators and policymakers.

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I. Introduction

Against which of these assets should one extend more credit in 2023: a suburban office or a warehouse facility leased to an e-commerce company? With the relative uncertainty surrounding work-from-home trends, the answer seems clear—but it may have been a tossup twenty years ago. This example illustrates the core motivation of our paper. Loan-to-value ratios (LTVs) are often viewed as a primary measure of underwriting standards in commercial real estate (CRE) lending.[1] We argue that variation in observed LTVs should be interpreted through the lens of the lender's (originator's) risk perceptions, which not only vary across collateral (property) types but also over time. What might be interpreted as "aggressively" high LTV, may, in fact, be optimal or justifiable given the property's riskiness and the business cycle. Theoretically, this idea is not new: Jaffee and Russell (1976) demonstrate that lenders may limit credit (i.e., require more "skin in the game") when it is hard to tell which borrowers are riskier, and Leland and Pyle (1977) provide a framework of firm financing with greater equity accompanying greater risk. Lenders' willingness to extend more credit should reflect perceived property risk and not only "loose" or "tight" credit conditions (which may also reflect lenders' risk tolerance or cost of capital).

Despite the clear role of risk perceptions, aggregate changes in LTV are often interpreted as changes in underwriting standards when it comes to measuring the aggressiveness of real estate lending. For instance, a 2010 Congressional Oversight Panel report by Elizabeth Warren et al. (2010) after the Global Financial Crisis (GFC) came to the following conclusion:

> *"The commercial real estate bubble [. . .] resulted in the origination of a significant amount of commercial real estate loans based on dramatically weakened underwriting standards. These loans were based on overly aggressive rental or cash flow projections [. . .], had higher levels of allowable leverage, and were not soundly underwritten."*

The culprits identified in the above quote are unrealistic cash flow forecasts and overly high LTVs, consistent with other assessments (e.g., Levitin and Wachter,

[1] Although the income-to-debt ratio and the debt service coverage ratios are also common qualifiers of CRE loan underwriting, we find that the empirical relationship between perceived property risk and LTV is stronger than the empirical relationship between property risk and these alternative ratios.

2013). While there is evidence to suggest that property income measures are, at times, inflated by commercial mortgage-backed securities (CMBS) originators (Griffin and Priest, 2023), the diagnosis of aggressive LTVs is a narrative that is more difficult to test, though it may often be conjectured. Jacob and Manzi (2005) describe what they believe to be lenders pushing the limit on LTVs in a trend toward "weaker lending standards," and Fabozzi, McBride and Clancy (2015) claim that this tendency was especially egregious in 2006 and 2007. Meanwhile, Wilcox (2012) and Wilcox (2018) argue that aggregate LTVs may not provide a faithful portrayal of underwriting standards.

We provide empirical evidence that, controlling for implied ex ante perceptions of property risk, as proxied by the implied volatility (IV) of individual properties, the average LTVs of securitized CRE loans in the period 2000–04 were only about 1.5 percentage points higher than the average LTVs in the post risk retention rule period of 2016–20. Likewise, average LTVs in 2005–07 were similar to those in 2008–15. Differences among epochs shrink further when we control for property cap rate (cash yield) spreads over the 10-year U.S. Treasury yield. Indeed, we find that credit rationing "frontiers" (i.e., maximum LTV thresholds) were most permissive in 2000–04, and they were most restrictive in 2005–07, coinciding with the peak of collateralized debt obligation (CDO) issuances. Importantly, credit rationing frontiers explain only a negligible fraction of LTV variation across epochs, while perceived property risk explains the lion's share.

Our main contribution is demonstrating that LTVs for securitized CRE loans, throughout different economic epochs from 2000 to 2020, were largely driven by perceived property risk and market fundamentals. We calculate implied volatility using a two-factor derivative asset pricing model, which allows for standard CRE mortgage contract provisions. In the model, IV is the asset's diffusion risk that rationalizes the loan's interest rate given its LTV, maturity, and amortization schedule, as well as the property's cap rate, the term structure of U.S. Treasury yields, and the mortgage market liquidity premium.[2] Our findings are consistent with tradeoff theories of optimal leverage (e.g., Leland, 1994), which imply that observed LTVs should decline with IV and cap rates. On its own, IV explains about 2/3 of the cross-sectional and time-series variation in LTV. Controlling for the cap

[2]We estimate liquidity premium on the CRE mortgage market as the effective yield spread between short-term AAA-rated tranches of CMBSs and U.S. Treasury securities with equivalent maturities.

rate and various (such as, property type and location) fixed effects helps explain an additional 10 percent of LTV variation. The residual time-series variation seems to be random. Our results are in line with Driessen and Van Hemert (2012) and Stanton and Wallace (2018), who find no evidence that underwriting practices in the CRE mortgage market deteriorated in the way that they did in the residential real estate mortgage market before the GFC. Our results are also consistent with the position in Wilcox (2012) and Wilcox (2018) that LTVs, on their own, may not be informative about aggregate loan underwriting standards.

From the collapse of the CRE market in the wake of the GFC, it is tempting to conclude that CRE loan leverage was overly aggressive before the crisis. However, after controlling for implied volatility, we find no evidence for abnormally high LTVs. What we do find is that risk perceptions were lower in 2003–07 than in any other epoch from 2000 to 2020. Hence, our findings suggest systematic shifts in perceived property risk as a compelling explanation for the growth in CRE lending in 2003–07, which fueled the subsequent CRE market decline. Moreover, to the extent that there was a failure in the CRE mortgage market in the run-up to the GFC, our findings may also indicate aggregate risk *misperceptions*.[3] Indeed, systematic misperceptions of risk would have led to more credit extended but also to CRE loan underpricing (i.e., low interest rates).

The paper is organized as follows. Section II provides theoretical background for conceptualizing the aggregate LTV as a dynamic variable that captures changes in systematic risk perceptions as well as other property and capital market attributes—even in the presence of frictions due to taxes and costs of default. Section II reviews the related academic literature and provides institutional details on the CMBS market. Section III describes our data. Section IV analyzes our implied volatility estimates and their relationship to other measures of property risk. Section V presents our main results, examining CRE loan LTVs over time with and without controls for determinants of optimal leverage, including (and especially) implied volatility. Section VI concludes.

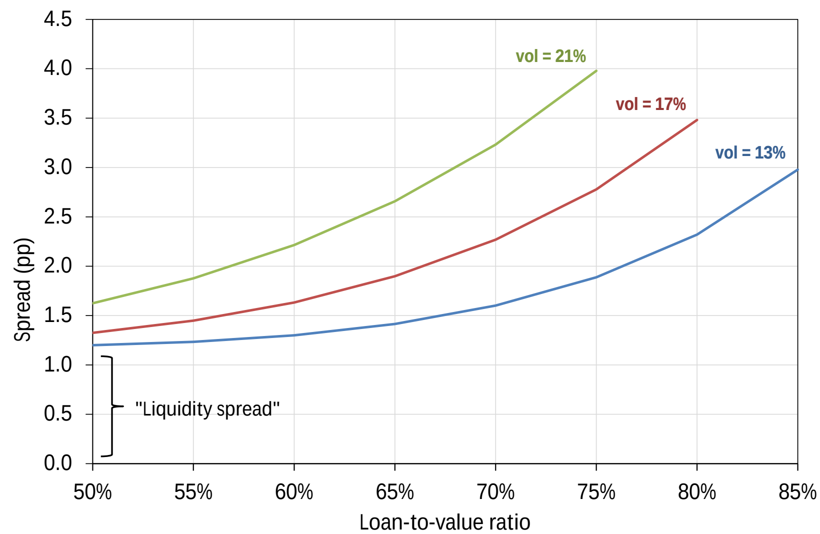
[3]There is an important distinction between aggressive loan underwriting and loan underpricing. In the former case, lenders knowingly undertake more risk than warranted by prudent practices. In the latter case, lenders falsely believe that they follow best underwriting practices.

II. Conceptual motivation and methodological overview

Mortgage provision in the primary market depends on the price of liquidity in the secondary mortgage market, competition among lenders, and the availability of capital in the credit market, all of which are held fixed. Figure 1 shows stylized mortgage offer curves for properties with different perceived risk.[4] The mortgage offer curves are truncated beyond a certain LTV because of credit rationing due to asymmetric information (Jaffee and Russell, 1976; Leland and Pyle, 1977) and dead-weight costs of default (Leland, 1994).

Figure 1. Stylized mortgage offer curves for properties with different risk

This figure shows stylized five-year zero-coupon mortgage offer curves for properties with different levels of perceived risk. Property risk is proxied by annualized asset volatility ("vol"). The vertical axis shows the mortgage spread over a zero-coupon five-year U.S. Treasury at which lenders would be willing to issue the mortgage loan given the loan-to-value ratio and the annualized asset volatility. The curves are computed using the Merton (1974) model and incorporate a liquidity spread, which represents the price of liquidity in the mortgage-backed securities market.



Given the mortgage offer curve specific to the property, the borrower's choice reduces to picking the LTV. In the Leland (1994) model, for example, the optimal borrower's choice is decreasing in the property's cap rate, increasing in the owner's

[4]In this stylized chart, we ignore other mortgage contract terms available to the borrower and commensurately priced by the lender, including maturity, interim coupon payments, and prepayment options. We do take such loan features into account in our empirical methodology.

tax rate, and decreasing in asset volatility. All else being equal, a model with rational agents yields equilibrium LTVs that decrease in asset volatility.

There are several key takeaways from the conceptual considerations above. First, time variation in aggregate LTVs may be entirely attributable to changing market fundamentals, rather than the tension between regulators and lenders or agency frictions within lending institutions. Second, in those instances where LTV limits are below the optimal leverage point for a sufficiently large number of borrowers, there would be clustering at the credit rationing frontier. Third, the frontier should decline with perceived property risk. Finally, although lenders and borrowers do not directly express or observe asset volatility, their risk assessment is implicit in the loan spread at which the contract is originated. In Figure 1 above, it is sufficient to know that a loan with a 67% LTV is priced at 2 percentage points above the five-year U.S. Treasury yield to conclude that the perceived property risk was roughly 17% in implied volatility.

A. Overview of empirical approach and results

Our null hypothesis is that the cross section of LTVs results from borrower demand in response to rational mortgage offer curves, akin to those depicted in Figure 1. Under the null hypothesis, it is consistent with prudent lending to provide an infinitely elastic supply of credit at any point on the curves. Risk misperception corresponds to lending using offer curves that are systematically lower or higher than the true asset volatilities, which could be detected ex post. By contrast, aggressive lending manifests ex ante as loans that would not normally be made (e.g., an 80% LTV loan with asset volatility of 21% in Figure 1). Hence, one could test for aggressive lending in a period like 2005–07 by examining whether the credit rationing frontier was higher during that period than at other times.[5]

In order to test the null hypothesis described above, and look for evidence of aggressive lending practices, we first estimate the implied volatility of each property underlying a sample of securitized CRE mortgages. The mortgage pricing model that we use to estimate IV captures various ex ante features of the deal, including the property's cap rate, the loan's term and amortization schedule,

[5]Note that changes in the credit rationing frontier could result from the influence of fundamental factors, such as a systematic change in dead-weight costs of default. Therefore, identifying an epoch with a higher rationing frontier does not constitute evidence for aggressive lending practices.

default and interest rate risk, and mortgage market liquidity.[6] Under the null hypothesis defined above, IV measures the lender's perceived property risk. Second, we identify the credit rationing frontier, as a function of IV, for four time periods in our sample. We confirm that, consistent with the null hypothesis, the frontier gradually declines with implied volatility in each of the four periods.

Next, we examine the rationing frontiers and empirically reject the narrative that lenders were more aggressive in the run-up to the GFC. We use ex ante fundamentals to explain the variation of LTVs across loans and over time, such as implied volatility, cap rate, and other property and market features that can influence mortgage offer curves and borrower demand for loans. We fit a censored (tobit) regression model for LTV, using the rationing frontier as upper bound, and conduct a counterfactual analysis. In particular, we fix the frontier over time and compare actual and estimated counterfactual LTVs to analyze the effect of changing frontiers. We find little evidence that shifts in the rationing frontier explain LTVs, which indicates that, even if such shifts are driven by changing underwriting standards, they have little effect on the distribution of LTVs.

What we do find is that the leading determinant of credit provision is implied volatility (perceived property risk). Importantly, this relationship is not mechanical. If borrowers randomly selected LTVs from the mortgage offer curves in Figure 1, then the only link between LTVs and risk perceptions would be through credit rationing, because the rationing frontier is more likely to be binding at higher IVs. This is not consistent with our result that shifts in the rationing frontier explain little of the distribution of LTVs. By contrast, if borrowers optimally choose LTVs to trade off costly default against the benefits of debt (e.g., lower taxes), then, consistent with our empirical findings, LTVs vary with property risk even if the rationing frontier is not a binding constraint.

B. *Literature on commercial real estate risk and mortgage implied volatility*

We contribute to an evolving understanding of CRE asset volatility. Previous analyses use aggregate data to study CRE price dynamics. Ciochetti et al. (2002) create a property value volatility index at the property type-census district level. Plazzi, Torous and Valkanov (2010) use quarterly averages at the metropolitan

[6] We also examine the possibility that prepayment options affect our results. We confirm that, because of the presence of large prepayment penalties in this market segment, they do not.

statistical area level for broad property types and apply the Campbell and Shiller (1988) price-dividend decomposition to better understand the characteristics of CRE rents, cap rates, and asset returns. They find that CRE returns are related to the local regulatory environment and population density, and that expected returns are related to factors such as local population, employment, and income growth as well as construction costs. Using property-level data, we find that many of these factors are important determinants of asset volatility.

Studies using property-level data have shed light on the magnitude of idiosyncratic asset volatility, that is, how much higher asset volatility is than can be inferred from indexes or other area averages. Plazzi, Torous and Valkanov (2010) calculate that, aggregated at the metropolitan statistical area-level, the standard deviation of CRE excess returns ranges from 3.7% to 6.1%, depending on the property type. By contrast, Downing, Stanton and Wallace (2008) estimate the asset volatility of CMBS loans using a two-factor Titman and Torous (1989) model. They find implied volatilities in excess of 20%—higher than our estimates for their sample period but similar to our post-GFC calculations. Sagi (2021) uses property-level data from the National Council of Real Estate Investment Fiduciaries to measure price appreciation volatility. He finds that the standard deviation of annual price appreciation volatility is about 13%.[7]

The mortgage pricing model we use to estimate implied volatility builds on an extensive body of literature that applies option theoretical methods for pricing mortgage debt. Some models stipulate a partial differential equation for property value that is solved using finite difference methods (Titman and Torous, 1989; Kau et al., 1995). Another popular method, and the one we employ, uses a binomial model for property valuation (Leung and Sirmans, 1990; Giliberto and Ling, 1992; Hilliard, Kau and Slawson, 1998; Ciochetti and Vandell, 1999). Similar to our pricing approach, many of these models incorporate default and prepayment options. However, while other models assume a single stochastic mean-reverting interest rate process similar to Cox, Ingersoll and Ross (1985), we model interest rates using multiple competing models. Furthermore, we include contractual

[7]Asset pricing models typically assume that the asset price follows a geometric Brownian motion, and thus the variance of cumulative price appreciation is linear in the length of the holding period. Consequently, as the holding period approaches zero, return volatility also approaches zero. By contrast, Sagi (2021) finds that volatility remains high even for short holding periods. He traces this phenomenon to transaction risk, finding that the return data fit well to predictions from a search model.

characteristics, such as interest-only versus amortizing payment schedules, and property attributes, such as the cap rate.

Our analysis is closest to that of Downing, Stanton and Wallace (2008). They also use a two-factor pricing model that prices the mortgage at par, albeit with the goal to examine the relationship between IVs and CMBS ratings. Similar to our approach, mortgage value in their model is a function of short rate dynamics and the property value process. By contrast, our model incorporates a richer set of loan and property characteristics, including property income and the length of the interest-only period. Finally, their analysis ends in 2006, while we also examine developments right before and after the GFC.

C. Evolution of the commercial mortgage-backed securities market

The CRE loans in our data set are CMBS loans: loans originated to be pooled within Real Estate Mortgage Investment Conduit trusts that issue MBSs. CMBSs allocate risk among different tranches: the tranches least exposed to credit risk typically receive investment-grade ratings, while the tranches that absorb credit losses first are often unrated. In the past two decades, various changes in the CMBS market affected both the cost of funding and the market for riskier tranches. In particular, the investor base of riskier tranches changed because of the rise and fall of CDOs as well as regulatory changes.

Before 2005, unrated tranches were usually held by a set of special ("B-piece") investors, who were involved in security design, performed due diligence, and selected the servicer responsible for handling delinquencies. In the time period between 2005 and 2008, it became common practice among CMBS issuers to repackage such tranches in CRE CDOs. Rating agencies, which made (overly) optimistic assumptions about the benefits of diversification, assigned favorable ratings to many CDO tranches. Another factor affecting CMBS markets before the GFC was a reduction in regulatory capital requirements, as both commercial bank and investment bank capital requirements for CMBS were reduced in 2004. Duca and Ling (2020) calculate that commercial bank capital requirements were reduced from 8% to 2%, while investment bank capital requirements were reduced from 6% to 3.7%, permitting much higher levels of leverage. As a result, the cost of funding decreased for both commercial and investment banks, making it easier for CMBS issuers to sell riskier tranches.

After the GFC, CMBS issuance stopped for several years and the CRE CDO market disappeared. In response to the crisis, regulators increased capital requirements for commercial banks at the end of 2010, by which time the major investment banks had been merged with commercial banks. Moreover, U.S. regulatory agencies proposed Regulation RR, requiring issuers of asset-backed securities to retain at least five percent of the credit risk, with the intention of ensuring that issuer incentives are aligned with those of investors.[8] Issuers may satisfy the risk retention requirement by holding a "vertical" piece of the issued security, which includes a portion of all tranches, a "horizontal" piece of the riskiest tranche, or a combination of the two approaches. While "qualified" CMBS issuances are exempt from Regulation RR, they are defined in a relatively conservative manner.[9] Consequently, the risk retention requirement is often binding for issuers (Flynn Jr, Ghent and Tchistyi, 2020).

Motivated by the substantive variation in the CMBS market and regulatory environment, we subdivide our sample period into the following four epochs.

1) 2000–04: "B-piece" investors retain the riskiest tranches of CMBSs.

2) 2005–07: CMBS issuers repackage portions of riskier tranches as CRE CDOs, part of which receive investment-grade ratings. Regulatory capital requirements associated with CMBS holdings decrease.

3) 2008–15: CMBS issuances plummet then gradually recover to pre-2005 levels.

4) 2016–20: The risk retention rule takes effect in December 2016.

These epoch boundaries are also consistent with the empirical distribution of CRE loan originations over time. Indeed, as Figure 2 shows, the number of loan originations exhibits clear cutoffs at the epoch boundaries we use.

[8] The risk retention rule was finalized in October 2014 and came into full effect in December 2016. See https://www.sec.gov/news/pressrelease/2014-236.html for more information.

[9] Regulation RR defines a qualifying CRE loan as a fixed-rate loan with a minimum maturity of 10 years and a maximum amortization period of 25 years. Lenders must document property income for at least the previous two years. The borrower's debt service ratio must exceed 1.25 for multifamily properties, 1.5 for leased properties, and 1.7 for all other loans. Furthermore, the combined LTVs of all loans on the property cannot exceed 70%, and the LTV of the first lien loan cannot exceed 65%.

III. Data construction and summary statistics

A. Securitized mortgage loan data collection

Our data consist of 58,127 securitized CRE loans from the year 2000.[10] The data are provided by Morningstar, which gathers information from public CMBS disclosures, including a rich set of loan and property characteristics. The Morningstar data include loans originated by a variety of institutions and are not dominated by a single underwriting approach. Many loans are originated by large U.S. banks, such as Bank of America and Citibank. The top ten originators also include large foreign banks, such as Deutsche Bank, Credit Suisse, and UBS. Non-depository institutions are a substantial part of the market, but no single such institution has a large market share.

The complete data set consists of 111,465 loans. For the purposes of our analysis, we drop loans missing key variables needed for our analysis and loans with problematic observations (see Appendix A for more details). Specifically, we drop loans that have missing or wrong data for key inputs such as the date of origination, the loan interest rate, whether the loan is interest-only or amortizing, and the date of maturity. We also drop non-fixed-rate and pari passu loans, which our model does not price, as well as agency CMBS loans because the agency guarantees would distort our implied volatility estimates. Finally, for analytical and expositional simplicity, we restrict our sample to single-property loans, which constitute the overwhelming majority of observations.

The data collection process described above results in a sample of 58,127 CRE loans. We present summary statistics for these loans in Table 1 and their corresponding cross-sectional distributions in Figure F2 of Appendix F. Loans vary widely in size, from \$2 million to over \$2.5 billion. LTVs are generally around 70%. The debt yield, the ratio of net operating income (NOI) to loan amount at origination, varies between 7% and 15%. The debt service coverage ratio (DSCR),

[10] Securitized CRE loans are only part of the overall CRE loan market. Black et al. (2017) compare CMBS loans in the Morningstar data with portfolio CRE loans reported by large U.S. banks on the FR Y-14 form. They find that these banks are more likely to hold riskier loans, such as construction loans, in their portfolio. Portfolio loans are also more likely to have floating interest rates, shorter terms, and lower LTVs than CMBS loans. From their results, it is not clear that securitized and portfolio CRE loans differ when one sets aside construction loans, which we exclude from our empirical analysis.

the ratio of NOI to debt servicing amount at origination, falls generally between 1.2 and 2.4. The vast majority (more than 80%) of loans are 10-year loans.

Table 1—Characteristics of sample commercial real estate loans

This table shows summary statistics for key characteristics of commercial real estate mortgage loans in our sample, containing fixed-rate, single-property loans securitized in non-agency commercial mortgage-backed securities (CMBSs). From left to right, the columns show the number of observations and the sample mean, standard deviation, as well as the 10^{th} and 90^{th} percentiles of variables in the cross section of sample loans. At the bottom of the table, the respective spread measures represent the percentage point yield spreads of sample loans over the 10-year zero-coupon U.S. Treasury yield and the value-weighted effective yield of the securities constituting the ICE BofA 0-to-3-year AAA U.S. Fixed-Rate CMBS Index.

	Count	Mean	SD	P10	P90
Loan amount ($1,000)	58,127	11,465	16,694	2,010	24,260
Loan term (months)	58,127	113	24	83	120
Amortization period (months)	58,127	312	110	0	360
Interest-only period (months)	58,127	21	35	0	60
Loan-to-value ratio	58,127	0.68	0.11	0.55	0.79
Debt yield	58,127	0.11	0.04	0.07	0.15
Debt service coverage ratio	58,127	1.69	0.71	1.18	2.32
Spread over 10-yr U.S. Treasury (pp)	58,127	1.85	0.75	0.92	2.78
Spread over 0-3-yr AAA CMBS (pp)	58,127	1.84	1.17	0.36	3.32

Figure 2 and Table 2 show the distribution of observations over time and across property types. The volume of loan originations steadily increased until the GFC, fell to almost zero in 2008, and gradually recovered after 2010. The most common property types are retail and multifamily, and also a large number of loans belong to the "other" category.[11] Hotel and industrial properties have the smallest frequency share in the sample.

We present summary statistics for the properties used as collateral for sample CRE loans in Table 3. The median property is nine years old and nearly fully leased. Properties vary widely in size, from 16 thousand to 24 million sqft. Some property-level variables are unevenly populated, mostly due to heterogeneous measurement and reporting standards across property types. For instance, information on the

[11]The "other" category consists of mini-storage and mixed-use properties representing a combination of property types such as a complex with both multifamily and retail property.

Figure 2. Annual number of sample commercial real estate loan originations over time

This figure shows the annual number of commercial real estate mortgage loan originations in our sample, color coded by time period (epoch). The sample contains fixed-rate, single-property loans securitized in non-agency commercial mortgage-backed securities. Epoch choice is explained in Section II.C.

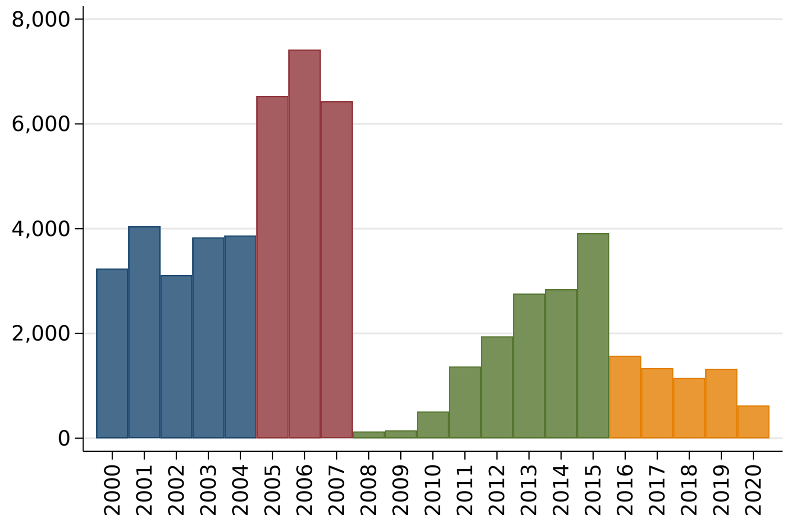


Table 2—Distribution of sample commercial real estate loans across property types

This table shows the absolute and relative frequencies of commercial real estate mortgage loans in our sample across different collateral property types. The sample contains fixed-rate, single-property loans securitized in non-agency commercial mortgage-backed securities.

Property type	Count	Share
Hotel	4,808	8.3%
Industrial	3,052	5.3%
Multifamily	18,972	32.6%
Office	9,308	16.0%
Other	5,903	10.2%
Retail	16,084	27.7%
Total	58,127	100.0%

lead tenant is not collected for multifamily properties because they have many small units, each leased to a different tenant.

In our data filtering, we drop loans with a debt yield less than 0.07 and DSCR less than 1.25. These lower bounds correspond to standard underwriting limits (i.e., lenders are reluctant to lend if the debt yield or DSCR is too low) and fall

Table 3—Characteristics of sample commercial real estate loan properties

This table shows summary statistics for key characteristics of the properties used as collateral for commercial real estate mortgage loans in our sample at the time of loan origination. The sample contains fixed-rate, single-property loans securitized in non-agency commercial mortgage-backed securities. From left to right, the columns show the number of observations and the sample mean, standard deviation, as well as the 10^{th} and 90^{th} percentiles of variables in the cross section of sample properties. The area of the property in square feet, as well as the derived variables occupancy rate and lead tenant share, are available only for industrial, office, retail, and most "other" property types. By commercial real estate market convention, the size of hotel and multifamily properties is measured by the number of units.

	Count	Mean	SD	P10	P90
Property value ($1,000)	58,127	17,524	28,014	3,070	36,000
Net operating income ($1,000)	58,127	1,158	1,771	214	2,335
Area (1,000 sqft)	33,562	112.45	151.08	15.99	240.02
Age (years)	52,569	14.26	15.77	1.00	35.00
Occupancy rate	31,929	0.94	0.10	0.83	1.00
Lead tenant area share	29,333	0.42	0.29	0.12	1.00
Lead tenant lease length (years)	29,832	15.98	240.45	2.17	16.25

around the 10^{th} percentile in our full sample of CMBS loans. When the debt yield and DSCR are very low, it may suggest that the property is not currently stabilized—even if it may be anticipated to be shortly. Since our model uses the property's underwritten cap rate as an input, including non-stabilized properties would distort our implied risk estimate. We are left with 48,468 observations, which we use throughout our empirical analysis in the paper.

B. *Adjustment for capital market liquidity dynamics*

One limitation of our mortgage valuation model is that it incorporates only two dynamic factors: the short interest rate process and the property value process. However, in practice, Christopoulos (2017) shows that mortgage pricing is also affected by a time-varying liquidity premium. Therefore, without appropriate correction, our model would attribute an increase in primary CRE mortgage rates due to a higher liquidity premium to increased credit risk, which would cause an upward bias in our volatility estimates.

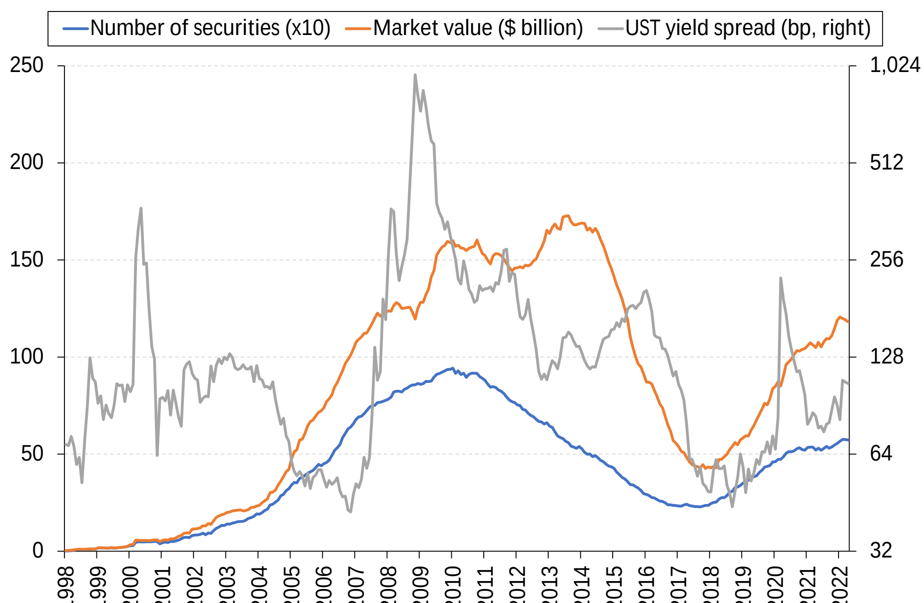
We take the liquidity premium into account by adjusting loan rates before the model-based property valuation step. Specifically, we create a monthly time series of the CMBS liquidity spread by taking the value-weighted effective yield of securities in the ICE BofA 0-to-3-Year AAA U.S. Fixed-Rate CMBS Index minus the yield of zero-coupon U.S. Treasury securities with the corresponding effective (i.e., option-adjusted) duration. We then adjust the mortgage rate for each loan by the prevailing liquidity spread as follows:

$$r_{adj} = r_{observed} - (\text{CMBS spread} - 120\text{bp}), \tag{1}$$

where 120 basis points is the median value of the CMBS yield spread defined above. Figure 3 shows the yield spread and the number and market value of the CMBSs with the shortest duration. Although this adjustment leaves a constant baseline level of liquidity premium embedded in mortgage rates, the remaining upward bias should permit *relative* comparisons of perceived property risk over time based on our model-implied volatilities.

Figure 3. Statistics for 0-to-3-year AAA fixed-rate commercial mortgage-backed securities

This figure shows quarterly aggregates from 1998 to 2022 for the commercial mortgage-backed securities (CMBSs) constituting the ICE BofA 0-to-3-Year AAA U.S. Fixed-Rate CMBS Index. UST yield spread is defined as the basis point (bp) difference between the value-weighted mean effective yield of the index constituents over the yield of the corresponding zero-coupon U.S. Treasury security with maturity equal to the value-weighted mean effective duration of the index constituents.



IV. Implied volatility estimation and diagnostics

Appendix B describes the two-factor model (with disaster risk) that we use to estimate implied asset volatility, which we then use as a measure for perceived property risk. The model ignores correlations between U.S. Treasury yields and property values. Although there is no standard way to measure property risk in the presence of market frictions and incompleteness, our implied volatility estimate is a sensible proxy measure of risk perception.[12]

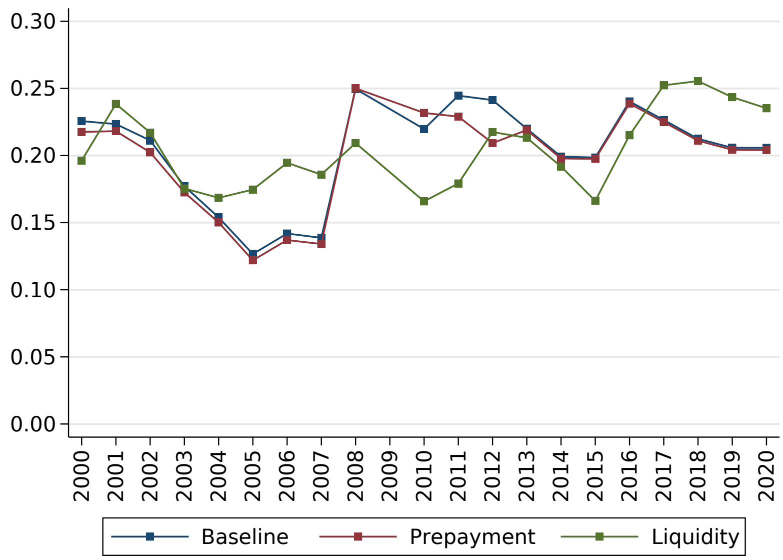
Figure 4 shows different implied volatility estimates based on our model. The first IV estimate makes no liquidity adjustments to mortgage rates and does not consider the effect of prepayment options. The second IV estimate allows for optimal prepayment in the presence of contractual penalties. Save for 2012, the presence of prepayment penalties makes little difference in implied volatility. This is because prepayment penalties, which are ubiquitous in the CRE mortgage market, are usually sufficiently punitive to render the value of a prepayment option second-order in mortgage valuation. Notably, given missing data problems (see Appendix A for further details), ignoring prepayment options in our model has the advantage of permitting a larger data set. The third IV estimate applies the liquidity adjustment to mortgage rates discussed in Section III.B and ignores prepayment options. Adjusting rates for mortgage market liquidity has a profound effect on the implied measure of property risk. Therefore, because of concerns raised earlier about risk mismeasurement, we use the liquidity-adjusted implied volatilities throughout our empirical analysis in the paper.

Figure 5 depicts the distribution of implied volatilities calculated using liquidity adjustments (and no prepayment options). The time series mean (median) is 20% (19%) and the standard deviation is 7.5%. The time variation in IVs is pronounced and corresponds to shifts in the entire distribution, which suggests that perceived property risk changes systematically over time. It is tempting to expect this time series variation to coincide with property market cycles, but that need not be the case because risk perceptions and liquidity on the credit market also affect the

[12] Contingent claims models, such as the one we use, assume that a risk-neutral pricing paradigm can be justified when one is able to replicate contingent claims. Clearly, this assumption does not hold in illiquid real estate asset markets. Conceptually, relying on the risk-neutral valuation methodology is similar to assuming the normality of unobserved shocks in a linear filtering problem. We can acknowledge the limitations of our methodology and attempt to provide validity or robustness tests for the approach, but our estimate is ultimately a proxy for, rather than actual, lender-perceived property risk.

Figure 4. Sample means of implied volatility estimates over time

This figure shows the cross-sectional means of the estimated model-implied volatilities of commercial real estate mortgage loans in our sample over time. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. The implied volatilities are estimated using the two-factor model described in Appendix B. There are three batches of estimates: a baseline batch without prepayment penalties or market liquidity adjustment, a batch with prepayment penalties, and a batch with market liquidity adjustment. The market liquidity adjustment process is explained in Section III.B.

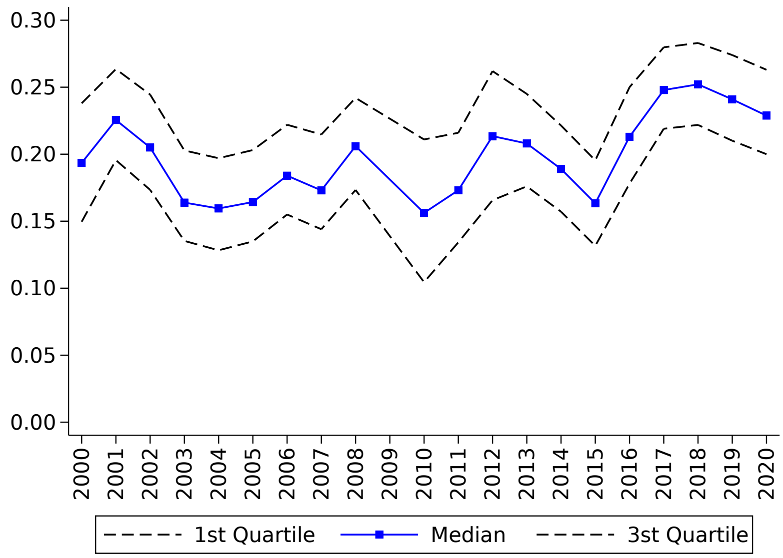


property market equilibrium. For example, during times of low perceived risk and high liquidity in credit markets, more properties meet lenders' and borrowers' criteria for financing. Hence, the effect of credit market cycles is also reflected in CRE loan terms and, ultimately, in our volatility estimates.

Indeed, the period with the lowest average IV is 2003–07, which coincides with the period of the greatest number of CMBS loan originations (Figure 2) and liquid credit markets. Meanwhile, IVs in 2008–10 are likely biased downward because lenders extended credit only to the safest properties as credit markets dried up (Figure 3). By contrast, the highest IVs come from 2001 and 2017–19, which are periods characterized by relative liquidity in credit markets. Such high IVs are a function of higher-than-average perceived property risk in the aggregate as well as an increased willingness by lenders and borrowers to finance riskier assets.

Figure 5. Sample quartiles of implied volatilities over time

This figure shows the cross-sectional quartiles of the estimated model-implied volatilities of the commercial real estate mortgage loans in our sample over time. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. For lack of observations, the quartiles cannot be estimated in 2009, when the commercial real estate mortgage market dried up.



A. *Structural determinants of implied volatility*

One potential critique of our use of implied volatility as a proxy for perceived property risk is the claim that lenders were risk insensitive when setting CRE loan spreads pre-GFC. In particular, our IVs might capture something other than property risk in the run-up to the GFC. For instance, pressure to originate for fees during the height of CDO issuances could have spurred competition for originating CMBS loans, resulting in exceptionally low mortgage rates, which do not accurately reflect the true risk of the underlying properties.

We address this critique, and validate the conjecture that implied volatility is related to structural determinants of property risk, by investigating the pre- and post-GFC drivers of IV and verifying whether relevant macroeconomic and property-level risk indicators contributed similarly to risk perceptions over time. Table 4 examines the pre- and post-crisis relationship between IV (the dependent variable) and various structural variables, such as state GDP, real estate sector GDP, unemployment rate, and income per capita as well as property size and age. Property and interacted state and time fixed effects are included. Property age, state GDP, and state employment rates are positively correlated with risk, consistent with the findings in Fisher et al. (2022) that urban density is associated with higher property market risk. Controlling for these variables, we find that property size and state income levels are negatively related to risk. Importantly, almost every coefficient that is significant post-GFC is also significant pre-GFC and has the same sign. If anything, the marginal effects of these variables on IV are stronger before the crisis than after it.

Table 4—Marginal effects of structural variables on implied volatility (%)

This table shows the estimated marginal effects of relevant local macroeconomic and property-specific variables on the estimated model-implied volatilities of commercial real estate mortgage loans in our sample. The sample consists of fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. The marginal effects are estimated on subsamples before and after the Global Financial Crisis (GFC), using a linear regression model with the logarithm of implied volatility as dependent variable. The model includes loan originator, property state, and property type-quarter of origination fixed effects. Standard errors are double clustered by state and quarter. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. Local macroeconomic variables are measured at a quarterly frequency and obtained from the Bureau of Economic Analysis. "GDP in sector" stands for the gross domestic product of the real estate industry.

	Pre-GFC	Post-GFC
100 × Log of state real GDP (USD mm)	0.008*	0.009**
100 × Log of state real GDP in sector (USD mm)	0.012	−0.002
100 × Log of state income per capita (USD)	−0.045**	−0.027***
State unemployment rate (%)	−0.128	−0.126***
Property Age (years)	0.022***	0.008***
100 × Log of property size (sqft)	−0.010***	−0.004***
100 × Log of property size (units)	−0.110***	−0.043***
Number of observations	25,493	15,869

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Additional analysis, not reported here, suggests no significant difference across IVs based on whether CRE loans were issued by large U.S. banks, smaller U.S. banks, foreign banks, nonbank lenders, or ex-post acquired or failed lenders. Overall, we find no empirical evidence that IVs were decoupled from property fundamentals before the GFC, as compared to the post-GFC period.

V. Loan-to-value ratios and risk perceptions

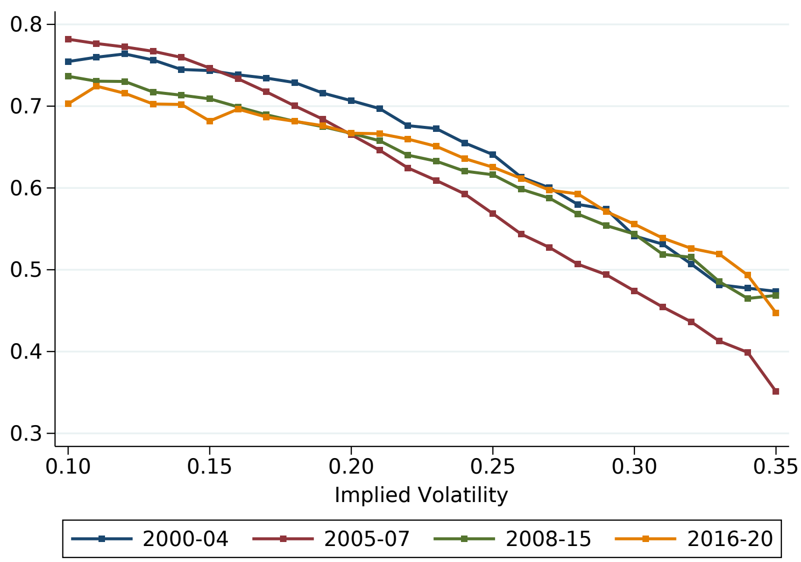
Figure 6 depicts average LTVs for each integer implied volatility "bucket" for loans originated in each of the four epochs described in Section II.C. As might be predicted by a tradeoff theory of optimal leverage, across all periods, LTVs exhibit a strong and negative relationship with implied volatility (IV). Indeed a univariate linear regression of IV against LTVs in our sample yields an adjusted R^2 that is an order of magnitude higher than a regression of IV against the other two common CRE mortgage underwriting metrics (debt service coverage ratios and debt yields). Fixing the level of risk, as proxied by implied volatility, it appears that LTVs were generally highest during the first epoch (2000–2004). The only exception comes from loans that were perceived to be relatively low-risk (below 15% implied volatility), for which the most aggressive period was Epoch 2 (2005–2007), when CDO issuance became prevalent. Interestingly, that same much-maligned epoch is associated with seemingly conservative LTVs for loans perceived as higher risk (higher than 20% volatility), and the post risk-retention period (Epoch 4) featured slightly higher LTVs than the prior Epoch spanning the period from the GFC to 2015.

It is important to emphasize that the strong relationship depicted in Figure 6 is not tautological. In a frictionless setting (a so-called Modigliani-Miller world), LTV would be arbitrary and plotting LTV against perceived property risk would yield no (or a random) relationship. By contrast, a theory of credit rationing (Jaffee and Russell, 1976; Leland and Pyle, 1977) or tax-bankruptcy tradeoffs (Leland, 1994) predicts a downward sloping relationship, which we observe in Figure 6.

Figure 6 suggests differences across time in leverage choice even after controlling for perceived property risk. The differences can arise simply from variations in the credit rationing frontier. They can also arise from differences in the property's cap rate or other, unobserved, differences specific to the property. Any variable that can alter optimal leverage can impact observed LTVs. These may include the local credit environment, the marginal tax rate of local investors, capital expenditure expectations that are not reflected in cap rates and other control variables. To further understand the differences across time in the supply and

Figure 6. Mean loan-to-value ratios across implied volatility bins and epochs

This figure shows the sample means of the loan-to-value ratios (LTVs) of commercial real estate loans that fall into a given integer bin of model-implied volatility and were originated in a given time period (epoch). The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. Epoch choice is explained in Section II.C. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B.



demand for credit against risky properties, one ought to attempt to control for such variables and examine how much each explains variation in LTV.

A. *Credit rationing frontier estimation and diagnostics*

In this subsection we investigate whether credit rationing limits (i.e., maximum LTV limits) set by lenders move in time. Our conjecture is that more aggressive lending practices would primarily expressed as increases to such limits. If the distribution of borrowers' demand for optimal leverage is constant, then applying a rationing limit would result in a truncated distribution of observed LTVs, and the observed LTV mean would move monotonically with the truncation point. Thus, once a rationing frontier is identified, a natural question to ask is how much variation in LTV is driven by changes to the frontier.

To begin, we first attempt to identify a lending "frontier" for various IV levels. In other words, what is the maximum credit level that lenders are willing to undertake for a certain risk perception? Given the scarcity of IVs at both extremes, we only do this for implied volatility levels between 5% and 40%. The existence of a frontier can be seen through a quick visual inspection of the data. For instance, Figure F3 shows clustering at around 80% LTV for implied volatility in the rough range of 0.05 to 0.2 across multiple time periods. Using quantile regression, we estimate the frontier as the 95$^{\text{th}}$ LTV percentile within each 1 percent implied volatility interval ("IV bucket") for each of the four time periods.[13]

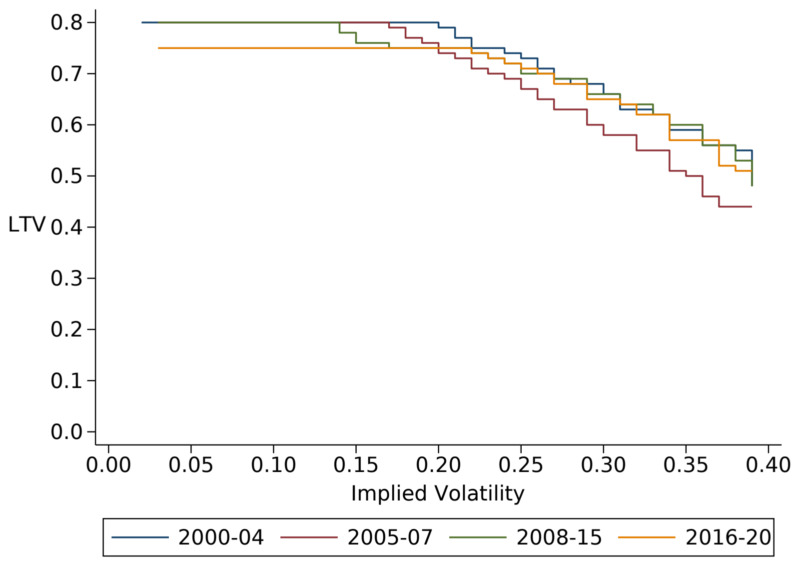
Figure 7 shows the calculated rationing frontiers by epoch across the implied volatility buckets. The 2005–2007 epoch stands out most in being visually different from, and generally lower than, the other three. Using a quantile regression, a pairwise comparison of marginal linear predictions across periods (Table 5) shows that the LTV frontier for this period is, on average, 4 percentage points lower than 2000–2004, and about 2-3 percentage points lower than both the 2008–2015 and 2016–2020 epochs. This may be surprising given a common perception that lending standards were looser in the period leading up to the GFC. These results suggest that, on the tail end of maximum loan extension, lending standards in the runup to the GFC were arguably *tighter* after controlling for risk perceptions. The least significant differences in frontiers occur between the two most recent epochs.

Overall, our analysis of rationing frontiers does not support a narrative that lenders were relatively more aggressive in 2005–2007. It is possible, however, that lenders' perceptions of property risk was systematically lower than what proved realistic, ex post. Such misperceptions would have unwittingly led to excessive provision of credit (e.g., extending an 80% LTV loan to a property judged to exhibit a property risk of 13% vol when the property was actually characterized by a risk of 21% vol). An alternative explanation is that originators in 2005–2007 did not care about risk when setting mortgage rates because property risk would accumulate in tranches that were subsequently placed in CDOs. While plausible at face value, this alternative is not supported by our analysis of determinants of

[13]The choice of LTV percentile where loans appear to cluster and denoting the frontier is robust to more sophisticated approaches, such as density discontinuity tests.

Figure 7. Credit rationing frontier estimates by epoch

This figure shows our credit rationing frontier estimates across time periods (epochs). The frontiers are estimated by fitting a quantile regression model for the 95$^{\text{th}}$ percentile of the loan-to-value ratios (LTVs) of commercial real estate loans that fall into a given integer bin of model-implied volatility and were originated in a given epoch. The estimation sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. Epoch choice is explained in Section II.C. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B.



IV in Section IV.A and leaves unanswered why credit rationing during 2005–2007 was substantially tighter for loans with IV greater than 20%.

B. Quantification of loan-to-value ratio determinants

Although there is no support for a looser credit rationing frontier during 2005–2007, it is still useful to understand whether movements in the frontier, which could be driven by imprudent extension of credit, have significant impact on the observed distribution of LTV. After all, based on the frontier analysis, one could argue that the first epoch, 2000–2004, was characterized by lending that was too permissive. Did that matter? In this section, we test this and, more broadly, seek to ask how much of the variation in LTV can be explained using economic fundamentals.

Table 5—Testing mean differences between rationing frontier estimates across epochs

This table shows the the results of statistically testing the mean differences between rationing frontier estimates across different time periods (epochs). The frontiers are estimated by fitting a quantile regression model for the 95$^{\text{th}}$ percentile of the loan-to-value ratios of commercial real estate loans that fall into a given integer model-implied volatility bin and were originated in a given epoch. The estimation sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. Epoch choice is explained in Section II.C. The implied volatilites are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. From left to right, the columns show the mean differences (Diff.), their standard errors (Std. err.), t-statistics, and corresponding p-values.

Frontier pair	Diff.	Std. err.	t-stat	p-value
2005–2007 vs. 2000–2004	−0.0410	0.0022	−18.64	0.0000
2008–2015 vs. 2000–2004	−0.0124	0.0016	−7.86	0.0000
2016–2020 vs. 2000–2004	−0.0245	0.0016	−15.22	0.0000
2008–2015 vs. 2005–2007	0.0286	0.0023	12.36	0.0000
2016–2020 vs. 2005–2007	0.0165	0.0023	7.09	0.0000
2016–2020 vs. 2008–2015	−0.0121	0.0018	−6.86	0.0000

Denote the demand for credit by the borrower for a given observed loan as LTV_i. The amount of credit that is observed to be extended is $cLTV_i = \min\{R(c_i, b(IV_i)), LTV_i\}$, where $R(c,k)$ is the rationing frontier in epoch c and implied volatility bin k, as identified in the previous section. We fit a censored linear regression (tobit) model to $cLTV$ of the form:

$$\begin{aligned} cLTV_i = & \max\big[0, \min\big\{R(c_i, b(IV_i)), \\ & \mu_{type} + \mu_{s,q} + o_i + \alpha(c_i)\mathrm{IV}_i + \beta_1 CRS_i + \beta_2 CRS_i{}^2 + \boldsymbol{m}_q\boldsymbol{\gamma} + \varepsilon_i\big\}\big] = \\ = & \max\big[0, \min\big\{R(c_i, b(IV_i)), x_i\beta + \varepsilon_i\big\}\big] \end{aligned} \tag{2}$$

where μ_{type} are property type fixed effects, $\mu_{s,q}$ are quarterly time fixed effects and state/county fixed effects, o_i are originator fixed effects, IV is implied volatility (with epoch-specific coefficients), CRS is the cap rate spread over the 10-year U.S. Treasury yield, and $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is the model noise term.[14] Additionally,

[14]The cap rate spread over the 10-year yield is a measure of the cap rate net of the risk-free rate. Leland (1994) shows that optimal LTV should decline with cap rate. The intuition is that, under the risk-neutral measure, all assets grow at the same rate (the risk-free rate) so an asset that reinvests income will grow more than an asset that distributes income. Correspondingly, a slower-growing asset is more likely to default at loan maturity. We use net cap rate because interest rates experienced a secular decline between 2000 and 2020, accompanied by a commensurately declining property cap rate.

$\boldsymbol{m}_q$ is a vector of quarterly macro-level variables that we include in model specifications without time fixed effects.

Table 6 shows the results of the tobit regression and the strong negative relationship between LTV and IV. Strictly on its own, and with a fixed slope coefficient, IV explains two thirds of the variation in LTV across time. Controlling for quarterly time fixed effects and property type fixed effects, 2005–2007 emerges as the epoch with the greatest sensitivity to IV. This relationship holds even as loan originator fixed effects for the 111 originators in our sample are taken into account, suggesting that individual originators, including those that systematically overstate property financials as documented in Griffin and Priest (2023), are not (on average) driving much of the variation in the LTV-IV relationship. The property's cap rate is highly significant and appears with the predicted negative sign (Leland, 1994), but only once we control for time, property, and state fixed effects. The CMBS yield spread of short-term AAA CMBS bonds over treasuries of equivalent maturity proxies for illiquidity in the mortgage market and explains a substantial portion of the time-series variation in the data. Viewed as a proportional cost of financing, the presence of illiquidity should negatively impact the choice of optimal leverage and this is consistent with the sign of its coefficient in Table 6.

Using the censored linear model coefficient estimates, we conduct a counterfactual analysis, investigating the effect of changing LTV determinants over time. In particular, we examine the effect of shifts in the rationing frontier across epochs on LTVs. To this end, we estimate counterfactual LTVs, denoted as $cLTV^*$, setting certain independent variables in the model constant over time. Formally, for each loan i, we estimate counterfactual outcomes

$$cLTV_i^* = \mathbb{E}\Big\{ \max\big[0, \min\{R(c_i^*, b(IV_i^*)), LTV^* \big] \mid cLTV_i, \hat{\boldsymbol{\beta}}, \hat{\sigma}_\varepsilon^2, \boldsymbol{x}_i^* \Big\}, \tag{3}$$

where $cLTV$ is the observable, $cLTV^*$ is the censored counterfactual, and LTV^* is the latent counterfactual LTV of the loan, $\hat{\boldsymbol{\beta}}$ and $\hat{\sigma}_\varepsilon^2$ are the coefficient estimates and the noise variance estimate from the 8th model specification in Table 6, and $\boldsymbol{x}_i^*$ is the vector of independent variable values we use for the calculation of a certain counterfactual scenario. Depending on the observed relation of $cLTV$ and the

Table 6—Estimation results of censored linear regression for the loan-to-value ratio

This table shows the estimation results of the censored linear regression model defined in Equation (2). The estimation sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. The dependent variable is the loan-to-value ratio, and IV stands for the model-implied volatility estimate for sample loans. The implied volatilites are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The columns show different model specifications with an expanding set of explanatory variables and fixed effects included. Standard errors are clustered by the quarter of loan origination and reported under the corresponding coefficient estimates in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
IV	-1.28							
	(0.05)							
2000–2004 # IV		-1.23	-1.21	-1.46	-1.40	-1.39	-1.38	-1.16
		(0.05)	(0.05)	(0.07)	(0.06)	(0.06)	(0.06)	(0.04)
2005–2007 # IV		-1.37	-1.32	-1.68	-1.64	-1.61	-1.64	-1.42
		(0.05)	(0.07)	(0.03)	(0.03)	(0.03)	(0.03)	(0.06)
2008–2015 # IV		-1.37	-1.34	-1.16	-1.29	-1.27	-1.26	-1.17
		(0.05)	(0.06)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)
2016–2020 # IV		-1.27	-1.25	-1.23	-1.31	-1.29	-1.28	-1.25
		(0.04)	(0.05)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)
Caprate spread			0.15	-0.07	-0.20	-0.40	-0.43	-0.14
			(0.19)	(0.13)	(0.12)	(0.12)	(0.12)	(0.16)
CMBS yield spread								-4.62
								(0.73)
UST 10yr yield								0.31
								(0.37)
Time (quarterly)				x	x	x		
Property type				x	x	x	x	x
Loan originator					x	x	x	x
Property state						x		
Property state × Time							x	
Property county								x
Generalized R^2	0.61	0.62	0.63	0.71	0.73	0.74	0.76	0.71
Number of observations	45,917	45,917	45,917	45,917	45,917	45,878	45,878	45,779

rationing frontier R, the expression in Equation (3) becomes

$$(4)\quad cLTV_i^* = \begin{cases} \max\left[0, \min\left\{R_i^*, cLTV_i + (\boldsymbol{x}_i^*\hat{\boldsymbol{\beta}} - \boldsymbol{x}_i\hat{\boldsymbol{\beta}})\right\}\right] & \text{if } cLTV_i < R_i, \\ \mathbb{E}\left\{\max\left[0, \min\left\{R_i^*, LTV_i^*\right\}\right]\right\} & \text{if } cLTV_i >= R_i, \end{cases}$$

where $R_i = R(c_i, b(IV_i))$ is the original rationing frontier, $R_i^* = R(c_i^*, b(IV_i^*))$ is the rationing frontier in the counterfactual scenario, and LTV_i^* is the latent counterfactual LTV of the loan, which follows the truncated normal distribution $\mathcal{N}_{TR}(\boldsymbol{x}_i^*\hat{\boldsymbol{\beta}}, \hat{\sigma}_\varepsilon^2)$ with lower bound $LB = R + (\boldsymbol{x}_i^*\hat{\boldsymbol{\beta}} - \boldsymbol{x}_i\hat{\boldsymbol{\beta}})$.

In Figure 8, we plot the mean LTV in each year of the original data set (Panel A) and for various counterfactual data sets (Panels B to F). The original data clearly exhibits a secular decline of LTVs over the sample period. However, this trend disappears in Panel B, which uses a counterfactual data set where IV for each loan is fixed at its sample mean (20%). Further setting the cap rate spread to its sample mean of 3.7% (Panel C) does not make much difference (consistent with the estimate in Figure 8, column 8). Fixing the CMBS yield spread does appear to reduce the time-series variation (Panel D) while fixing the US treasury 10-year treasury yield makes little impact (Panel E). Perhaps most importantly, fixing the rationing frontier to correspond to the first epoch (2000–2004) appears to have little impact on distribution of LTV time series means (Panel F). Recalling that the first epoch features the most permissive rationing frontier and the second the least, one would expect to see a large difference in the 2005–2007 data when moving from Panels E to F. This result suggests that shifts in the rationing frontier have little effect on the distribution of LTVs.

Table 7 compares the annual counterfactual means throughout the four epochs and shows that they are statistically distinct. This means there is still statistically significant remaining variation between the epochs after controlling for IV, cap rate, CMBS spread, U.S. Treasury yield, and changes in the credit rationing frontier. That said, the remaining time variation does not clearly fall into a pattern coinciding with macro events and may correspond to unobserved demand factors in the market for CRE loans. Moreover, after controlling for these influences, the differences in mean LTV are economically small: Less than 3% across all three epochs in Regression 6 of Table 7. Even more interesting is that the unexplained average difference between Epochs 1 and 4 as well as the difference between Epochs 2 and 3 are approximately 1% or less.

It is clear from the adjusted R^2 in Table 6 as well as the counterfactual LTVs in Figure 8 that the first order determinant of LTV in the sample is the perceived property risk. To provide a sense of the marginal contribution that the explanatory variables provide each sample year, we sequentially decompose the variance of

Table 7—Actual and counterfactual means of loan-to-value ratios across epochs

This table shows the means of the actual (Column 1) and counterfactual (rest of the columns) loan-to-value ratios of commercial lean estate loans in the sample across time periods (epochs). The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. The counterfactual loan-to-value ratios are estimated by applying Equation (3) and using the 8^{th} censored linear model specification in Table 6. Each regression, (2)-(6) incrementally fixes the values of various explanatory variables. US10 stands for the 10-year zero-coupon U.S. Treasury yield, while IV and CRS stand for model-implied volatility and capitalization rate spread over the US10, respectively. CMBS stands for the market liquidity spread defined in Section III.B. Robust standard errors are reported in parentheses. At the bottom, F-statistics are reported for the joint mean equality tests across epochs 1 to 4 and epochs 2 to 4, respectively.

Epoch	(1)	(2)	(3)	(4)	(5)	(6)
2000–2004	68.8	69.4	69.8	69.7	69.1	69.1
	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)
2005–2007	68.7	67.1	68.1	67.1	66.5	66.8
	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
2008–2015	66.8	66.2	66.6	67.3	67.6	67.8
	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)
2016–2020	62.8	67.6	68.2	67.8	68.1	68.4
	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)
IV = 20%		x	x	x	x	x
CRS = 370bp			x	x	x	x
CMBS = 120bp				x	x	x
US10 = 3.2%					x	x
Frontier set to epoch 1 level						x
Wald F Stat. of 1–4 equality	587.7	430.4	438.9	403.2	368.1	300.1
Wald F Stat. of 2–4 equality	711.4	109.1	224.1	20.9	175.9	160.9
Number of observations	47,616	45,779	45,779	45,779	45,779	45,779

LTV in the sample into components of the form

$$\text{comp}_{n,t} = Cov\Big(cLTV_t,\ cLTV_t^*|_{(x_1,\ldots,x_{n-1})=(\bar{x}_1,\ldots,\bar{x}_{n-1})} - cLTV_t^*|_{(x_1,\ldots,x_n)=(\bar{x}_1,\ldots,\bar{x}_n)}\Big)\Big/ Var\big(cLTV_t\big), \tag{5}$$

where $cLTV_t^*|_{(x_1,\ldots,x_n)=(\bar{x}_1,\ldots,\bar{x}_n)}$ is the subsample of counterfactual LTVs in year t, generated by fixing variables x_1 to x_n at their sample means.[15] The first component is simply

$$\text{comp}_{1,t} = Cov\Big(cLTV_t,\ cLTV_t - cLTV_t^*|_{x_1=\bar{x}_1}\Big)\Big/ Var\big(cLTV_t\big),$$

[15]The variable x_i can be viewed as a vector, or "block", of explanatory variables whose contribution to the variance is sought. This is roughly similar to an ANOVA decomposition, although components can be negative because the covariance is restricted to a subsample.

and is essentially the R-squared of regressing $cLTV_t$ against x_1 (because variation in the second term of the covariance only arises from variation in x_1). The last component, $\text{comp}_{n,t}$, is the proportion of variance that cannot be explained with $x_1, \ldots, x_n$.

Panel A of Figure 9 depicts the decomposition when x_1 is IV and x_2 corresponds to all remaining explanatory variables. Implied volatility explains between 40% and 70% of LTV variance in any given year. The incremental contribution of all other independent variables to explaining variance is no more than 20% (in 2000) and averages 10%. Of the non-IV independent variables, cap rate spread, originator fixed effects, and geographic fixed effects are the most important sources of variation. Their contributions are depicted in Panel B of Figure 9 and appear to be most pronounced pre-GFC and especially during the first epoch (2000–2004). One might be troubled by the fact that originator fixed effects capture variation in LTVs in the early part of the sample. A mitigating observation is originator effects substantially decline in 2005–2007 relative to 2000–2004. As suggested by earlier analysis, variation in the rationing frontier over time plays little role.

VI. Conclusions

Theory (e.g., Leland, 1994) suggests that optimal leverage choice depends on asset and market-specific factors. One of the most important of these is the risk of the underlying asset. We demonstrate that the single most important determinant of observed loan-to-value ratios (LTVs) in securitized commercial real estate loans is perceived property risk, as measured by implied volatility. On its own, perceived risk explains roughly two thirds of cross-sectional and time-series variation in LTVs. We find that other theoretically-motivated market-level and asset-specific fundamentals also drive observed choices of LTVs, albeit explaining no more than an additional (roughly) 10% in variation.

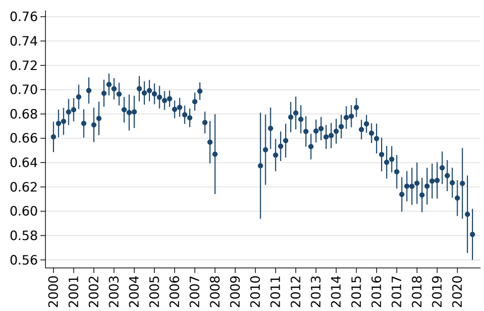
While LTVs have declined throughout our sample period (from 2000 until 2020), this secular decline disappears once one controls for the fundamental factors mentioned above. Remaining time variation does not appear to reflect any market trends. This is significant because LTV is commonly seen as an important metric of lending standards and often referenced in regulations (e.g., DiSalvo and Johnston, 2018). We find some evidence that aggregate LTVs contains information about lending standards through shifting maximum LTV criteria. However, the shifts we identify do not support the narrative that lending standards were least restrictive during the run-up to the GFC. Moreover, there is little evidence changes in maximum LTV criteria materially impacted the distribution of LTVs in our sample. We do, however, find that the key driver of LTV choice in our data set, average perceived property risk, was significantly lower in the five years leading to the GFC than at any other five year period in the past 20 years. This raises the possibility that lenders and borrowers systematically underestimated property risk between 2003–2007, leading to more-than-warranted credit being extended against commercial real estate and exacerbating the subsequent market downturn.

To the extent that ex post poor lending outcomes can be traced to systematic risk misperception, our work can be used to motivate the use of aggregate measures of market-specific risk perceptions, like loan-implied volatility, by regulators and policymakers.

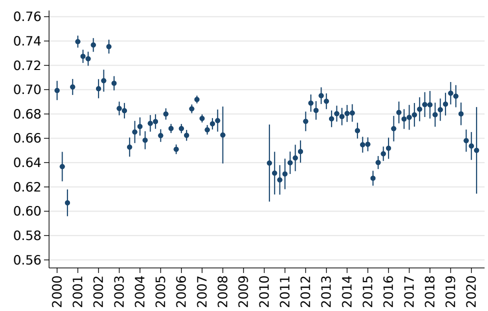
Figure 8. Means of actual and counterfactual loan-to-value ratios over time

This figure shows the means of the actual (Panel A) and counterfactual (rest of the panels) loan-to-value ratios of commercial lean estate loans in the sample over time, with 99% confidence intervals. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. The counterfactual loan-to-value ratios are estimated by applying Equation (3) and using the 8^{th} censored linear model specification in Table 6. Panels B to F incrementally fix the values of various explanatory variables. UST stands for the 10-year zero-coupon U.S. Treasury yield, while IV and CRS stand for model-implied volatility and capitalization rate spread over the UST, respectively.

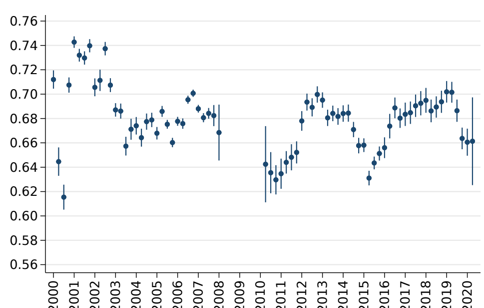
(a) Actual sample means



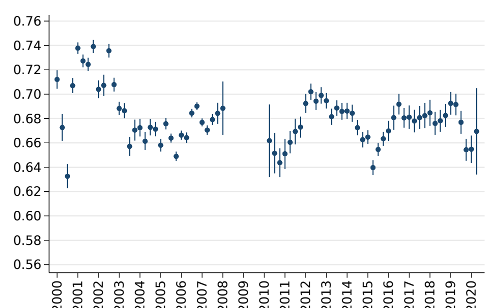
(b) Setting IVs to 20%



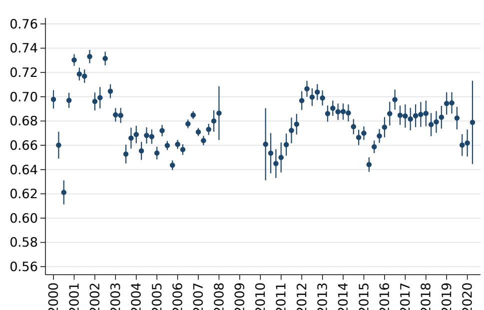
(c) Setting IVs to 20% and CRSs to 370bp



(d) Setting IVs to 20%, CRSs to 370bp, and CMBS spread to 120bp



(e) Setting IVs to 20%, CRSs to 370bp, CMBS spread to 120bp, and UST to 3.2%



(f) Setting IVs to 20%, CRSs to 370bp, CMBS spread to 120bp, UST to 3.2%, and fixing rationing frontier at Epoch 1 level

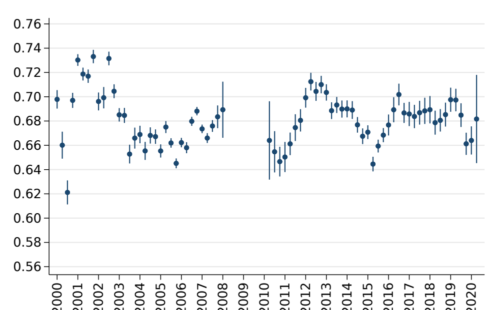
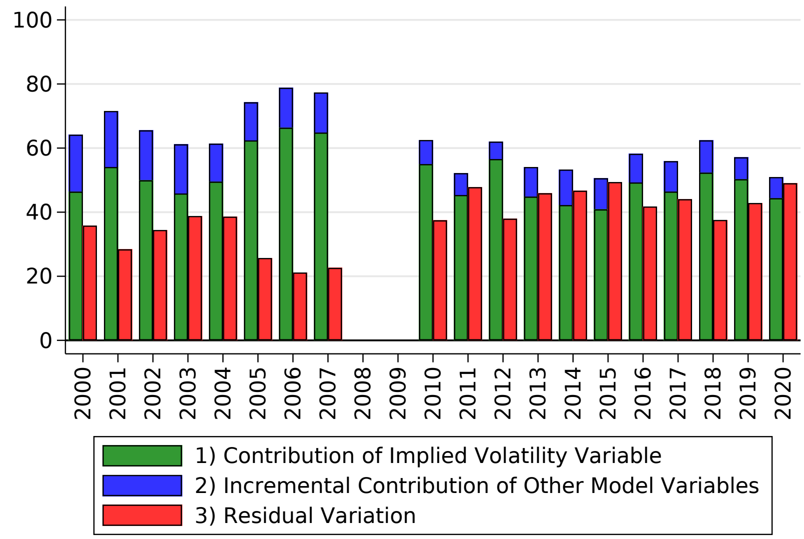


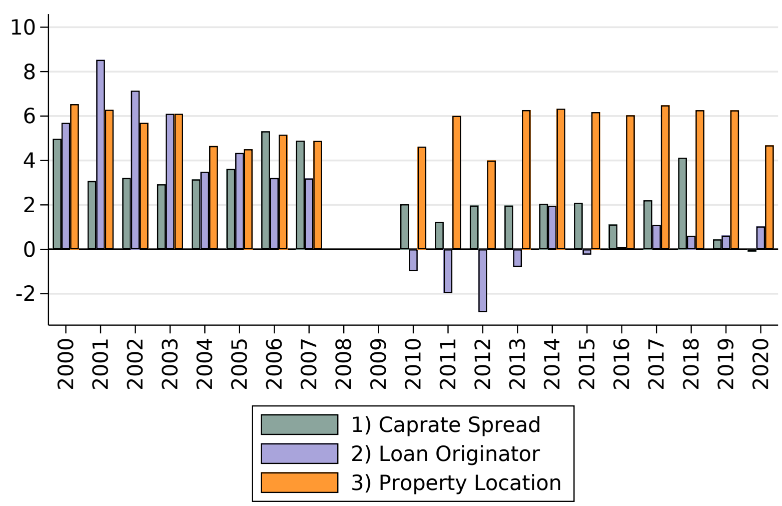
Figure 9. Model-based variance decomposition of loan-to-value ratios over time

This figure shows the model-based variance decomposition of the loan-to-value ratios (LTVs) of commercial lean estate loans in the sample over time. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. The variance decomposition applies the methodology described in the text (see Equation (5)), measuring the contribution of each variable to the variance of LTV in the context of the censored linear model defined in Equation (2). More specifically, LTV is modeled using the 8^{th} censored linear model specification presented in Table 6. Panel A decomposes the variance of LTV into the contribution from implied volatility, other model variables, and residual variation. Panel B presents the variance contribution of the three most relevant model variables other than implied volatility.

(a) Variance attributable to implied volatility and other model variables



(b) Variance attributable to selected other model variables



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Appendix A: Data construction process

The initial data set consists of 171,421 loans.

- We eliminate loans in deals originated by Freddie Mac or Fannie Mae. Both institutions are heavily involved in affordable housing, senior housing, and other subsidized projects. The pricing of such loans may not fully reflect the market perception of risk. Not all of their loans are for subsidized projects, but to our knowledge there is no efficient way to distinguish them from others.

- We eliminate loans with missing key variables such as origination date, maturity date, coupon rate, original loan amount, underwritten NOI, and origination LTV. We also drop loans with unrealistic values for these variables.[16]

- We remove a small number of loans with both defeasance and yield maintenance penalties (our model is not set up to take more than one prepayment penalty), more than three call protection options, and loans with an ambiguous call protection designation, such as "prepayment penalty."

- We have a number of loans for which the call protection lengths plus seasoning do not add up to the loan term. We have 326 such loans which undershoot the loan term and 6,908 which overshoot the loan term. When they undershoot, we simply extend the last call protection period. When they overshoot, we start subtracting from the last call protection type, then the second to last, then the first. We end up with 3,506 loans for which the last call protection type ends up getting completely removed. We only calculate IV for these loans in the batches excluding prepayment.

- Our model assumes that dividends, relative to property value, are constant for the life of the loan. In reality, some loans are for renovation purposes or fund other projects that would result in NOI increases. Such loans may include projected cash shortfalls during the beginning of loan life, relative to

[16]We require annual interest rates to be between 1% and 25%, loan amounts to be at least \$10,000, LTVs to be less than 100% and greater than 10%, and first year projected NOI at origination to be less than the property value.

the required debt service. We keep only loans whose debt service coverage ratio is greater than 1.25, and whose debt yield is greater than 7%.

- Our model also assumes that the property is collateral for a single loan only. Multiple forms of debt create potentially complicated dynamics between different creditors. We drop 1,850 multi-property loans as well as a number of loans in pari passu deals.
- We drop a small number of loans with maturities longer than 12 years as well as those originated before 2000 and are missing zip code data.
- NCF isn't as well populated as NOI, so we multiply NOI by a factor of 0.94 to match the average NCF (this is only done for our implied volatility calculations).

Appendix B: Implied volatility model

B1. Interest rate process

Gupta and Subrahmanyam (2005) run a horse-race among several prevalent pricing models and find that the pricing accuracy of one-factor models is comparable to that of other, more complicated, models. We use two of the models they examine: the Hull and White (1990) (HW) and Black and Karasinski (1991) (BK) models. These are some of the most commonly employed term-structure models for pricing interest rate derivatives in practice. We modify both the HW and BK models so that no more than one tree branch can be above 10% or below zero.[17] This is done to ensure that risk neutral probabilities for the property price model are positive at property diffusion volatilities as low as 3%. We note that, during our sample period, forward rates for a one-year zero coupon U.S. Treasury bond never exceed 7.5% or fall below 0%. Our bounds therefore likely reflect market perceptions for the possible range of interest rates during the life of originated mortgages in our data set.

To calibrate each month's term structure model, yield data are obtained for nominal zero coupon bonds with maturities ranging from one to twelve years.[18] Data for swaptions with exercise maturity of one year, the most liquid contracts, are obtained from Eikon for tenors (underlying swap maturities) of one, five, and ten years. Each month, we fit a HW and a BK model to the data and select the one that best fits the swaptions data.[19] Table B1 summarizes percentage price accuracy across the monthly term structure models that we estimate.

Our term structure models are generally accurate. Periods where the pricing error exceeds 5% are concentrated between 12/2008 to 03/2009, 09/2011 to 12/2012, 02/2016 to 11/2016, and after 03/2020. The BK model seems to be the

[17]This is achieved as follows. If the conventional HW or BK tree is consistent with the bounds, we employ it. Otherwise, we truncate all branches beyond the first that cross the bound by setting their transition probabilities to zero. At any node for which a branch probabilities is set to zero, we solve for the remaining branch probabilities by enforcing the node's expected interest rate to equal the quantity implied by the underlying mean-reverting process. The resulting rate volatility at edge nodes is generally distinct from the constant volatility elsewhere in the tree.

[18]The data are taken from the Federal Reserve: https://www.federalreserve.gov/data/nominal-yield-curve.htm. For the short end of the term structure, we use the 3-month U.S. Treasury constant maturity yield obtained from the St. Louis Fed.

[19]Both HW and BK models can be made to fit any arbitrary term structure of zero coupon bonds.

Table B1—Term Structure Model Precision

This table shows the accuracy of 277 "best-fitting" term structure models, which are estimated each month from 06/1997 to 06/2020 using zero coupon bonds and one-year swaption prices. The two models used for estimation are Hull and White (1990) and Black and Karasinski (1991) models. Each data point corresponds to the root of the weighted mean of squared pricing errors (i.e., percentage accuracy) from a single month's term structure model.

Statistic	TS Model precision
Mean	0.0202
SD	0.0439
P1	0.0001
P5	0.0003
P10	0.0007
P25	0.0015
P50	0.0042
P75	0.0149
P90	0.0534
P95	0.1146
P99	0.2322

better performer in roughly 2/3 of cases, and nearly exclusively so between 2007 and 2015.

B2. Property value process

Property value follows a binomial process similar to Cox, Ross and Rubinstein (1979), but modified to incorporate time-varying short-term interest rates and a possible catastrophic fall in value that triggers immediate default. The latter modification is motivated by the actual distribution of creditor losses. Without the possibility of a sudden (discontinuous) drop in property value, optimal exercise of the default option tends to predict relatively small loan losses relative to what is observed in practice. The "catastrophic" property-level event is Poisson distributed and assumed to arrive with annualized intensity of λ. The event is assumed to permanently reduce the property's value to zero, and the rate λ is calibrated to our loan pool to match historical CRE loss given default (LGD) rates of 30-35%.[20]

[20]This range is based on estimates by Esaki, L'Heureux and Snyderman (1999), Ciochetti (1997), and Curry, Blalock and Cole (1991). One could potentially model a distribution of catastrophic losses, but it is unclear to what extent this would make our results more reflective of lender perceptions versus a fixed LGD value. See Appendix D for further methodology.

Let σ be the annual volatility of the value of the property. To modify the binomial model of Cox et al. to accommodate the hazard, we divide the usual "up" and "down" states by $(1 - \lambda\Delta t)$ for each increment of time, Δt:

$$u = \frac{e^{\sigma\sqrt{\Delta t}}}{1 - \lambda\Delta t} \qquad\qquad d = \frac{e^{-\sigma\sqrt{\Delta t}}}{1 - \lambda\Delta t}.$$

The property value changes by a factor of u or d. This keeps the expected price appreciation of the property, under the risk-neutral measure, independent of the value of λ and thus independent of the idiosyncratic event. It also has the virtue of setting the Arrow-Debreu prices of "up" and "down" non-disaster states equal to $(1 - \lambda\Delta t)$ times their usual values in the Cox model:

$$\pi^{(u)}_{j,t,k} = \frac{e^{r_{k,t}\Delta t} - d(1 - \lambda\Delta t)}{u - d} \qquad\qquad \pi^{(d)}_{j,t,k} = \frac{u(1 - \lambda\Delta t) - e^{r_{k,t}\Delta t}}{u - d},$$

where we denote property state j, time t, and continuously compounded short interest rate state k obtained from the term-structure model. We assume that a one-period binomial "up" or "down" move in the property price process is uncorrelated with the one-period short-term interest rate process.

Commercial properties generate income for their owners, which we incorporate by assuming that the property pays a constant annual "dividend" rate δ_t corresponding to the property's ratio of net cash flow (NCF) to total appraised property value at the time of mortgage origination.[21] We include the dividend in our property value formulation, with the exception of origination, where property value is equal to appraised value.[22] We define "up" and "down" "cum-dividend" property value $V_{j,t+1}$ for all non-origination periods $t \in \{1, ..., T-1\}$ recursively as follows:

$$V_{j_u,t+1} = uV_{j,t}(1 - \delta_t\Delta t) \qquad\qquad V_{j_d,t+1} = dV_{j,t}(1 - \delta_t\Delta t).$$

[21]We use NCF instead of net operating income (NOI), as NCF subtracts CapEx and CapEx reserves and may be a better measure of actual cash flow.

[22]We assume that the property value at mortgage origination is calculated *after* cash flow from operations is distributed.

Mortgage terms comprise the LTV (or, equivalently, the amount borrowed), time to maturity, and the amortization schedule. Together with a complete specification of the property and interest rate model parameters, the mortgage terms imply a fair-market mortgage rate that can be calculated by setting the present value of the mortgage obligation to the amount borrowed. In practice, contract mortgage rates are observed but the underlying property volatility, σ, is unobserved. We therefore solve for the implied asset volatility that sets the present value of the mortgage obligation to the amount borrowed given the observed mortgage rate.

We denote property value $V_{j,t,k}$ ($V_{j,t} = V_{j,t,k}$ since property value is independent of interest rate movement), and corresponding equity and debt values $E_{j,t,k}$ and $D_{j,t,k}$. We allow for interest-only or amortizing mortgage payment schedules (or a combination of these). We denote the remaining mortgage balance B_t, and fixed mortgage payment or coupon c_t (B_t remains constant during an interest-only period). As in Cox et al., it is easiest to define our model by working backwards from maturity. Similar to Merton (1974), we define borrower equity and debt at maturity T:

$$E_{j,T,k} = \max(0,\ V_{j,T} - (B_T + c_T))$$
$$D_{j,T,k} = \min(V_{j,T},\ B_T + c_T).$$

These follow from the assumption of "ruthless" default: the borrower will default if the property value falls below the debt value. It is worth emphasizing that the Modigliani-Miller value additivity holds: $D_{j,t,k} + E_{j,t,k} = V_{j,t,k}$.[23] In other words, we assume no dead-weight cost of default.[24] Note that there is no prepayment or dividend payment at maturity. For each non-maturity and non-origination period $t \in \{1, ..., T-1\}$, the following equations determine the

[23]See Appendix C for proof that M&M holds at all periods t.

[24]We opt to ignore dead-weight cost of default because we do not correspondingly model tax benefits of debt or investors with heterogeneous private values; if mortgage debt only came with costs and no benefits, no rational investor would finance a property with mortgage debt.

borrower's value of equity and debt:

$$E_{j,t,k} = \max\left(0,\ \delta_t \Delta t V_{j,t,k} - c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{E}_{t+1}\right],\ V_{j,t,k} - c_t - B_t - P_{j,t,k}\right)$$

$$D_{j,t,k} = \min\left(V_{j,t,k},\ c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right],\ c_t + B_t + P_{j,t,k}\right),$$

where $\mathbb{E}_{j,t,k}[\tilde{E}_{t+1}]$ and $\mathbb{E}_{j,t,k}[\tilde{D}_{t+1}]$ represent risk-neutral expected values for equity and debt and $P_{j,t,k}$ is the prepayment penalty. The terms in each equation represent values for default, continuation, and prepayment options, respectively. For further clarity, risk neutral expected values for $X \in \{E, D\}$ are defined as follows:

$$\begin{aligned} \underset{j,t,k}{\mathbb{E}}\left[\tilde{X}_{t+1}\right] = \pi^{(u)}_{j,t,k} & \left[i^{(u)}_{k,t} X_{j_u,t+1,k_u} + i^{(m)}_{k,t} X_{j_u,t+1,k} + i^{(d)}_{k,t} X_{j_u,t+1,k_d}\right] + \\ \pi^{(d)}_{j,t,k} & \left[i^{(u)}_{k,t} X_{j_d,t+1,k_u} + i^{(m)}_{k,t} X_{j_d,t+1,k} + i^{(d)}_{k,t} X_{j_d,t+1,k_d}\right] \end{aligned}$$

with $i^{(u)}_{k,t}$, $i^{(m)}_{k,t}$, and $i^{(d)}_{k,t}$ being interest rate up, middle, and down state probabilities. In the initial origination period $t = 0$, we take the values of equity and debt to be their continuation values: $E_0 = (e^{-r_0\Delta t})\,\mathbb{E}_{0,0,0}[\tilde{E}_{t+1}]$ and $D_0 = (e^{-r_0\Delta t})\,\mathbb{E}_{0,0,0}[\tilde{D}_{t+1}]$ (we assume no mortgage coupon payment at origination and no dividend as noted above). After inputting all given mortgage values, we calculate an annual implied volatility figure σ such that D_0 matches the given contract loan amount.

Prepayment rules are specified in the mortgage covenant, and usually vary by time period in the mortgage. For instance, a common feature is a prepayment lockout of several months when prepayment is not allowed, followed by a lengthy period where prepayment is allowed but with penalties (usually defeasance or yield maintenance), followed by another shorter period where prepayment is allowed without penalties (the "open" prepayment period). These periods sum to the length of the mortgage. We model this as follows: we remove the prepayment option during the lockout period, explicitly model defeasance or yield maintenance during penalty periods,[25] and set $P_{j,t,k}$ equal to zero during open periods.

[25]See Appendix E for exact methodology.

Appendix C: Proof of Modigliani & Miller additivity

We would like to show that the Modigliani–Miller additivity $E_{j,t,k} + D_{j,t,k} = V_{j,t,k}$ holds for all $t \in \{0, ..., T\}$.

Part 1 of Proof

We begin by demonstrating that $V_{j,t,k} = \delta_t \Delta t V_{j,t,k} \ + \ (e^{-r_{k,t}\Delta t})\, \mathbb{E}_{j,t,k}[\tilde{V}_{t+1}]$ for $t \in \{1, ..., T-1\}$. We redefine the following (note that $V_{j,t} = V_{j,t,k}$ since property value is independent of interest rate movement):

$$\left.\begin{aligned} V_{j_u,t+1,k} &= uV_{j,t,k}(1-\delta_t\Delta t) \\ V_{j_d,t+1,k} &= dV_{j,t,k}(1-\delta_t\Delta t) \end{aligned}\right\} \text{for } t \in \{1, ..., T-1\}.$$

Now we use these to show that $V_{j,t,k} = \delta_t \Delta t V_{j,t,k} \ + \ \left(e^{-r_{k,t}\Delta t}\right) \mathbb{E}_{j,t,k}\left[\tilde{V}_{t+1}\right]$:

$$\begin{aligned}
&\delta_t \Delta t V_{j,t,k} \ + \ \left(e^{-r_{k,t}\Delta t}\right) \mathop{\mathbb{E}}_{j,t,k}\left[\tilde{V}_{t+1}\right] \\
&= \delta_t \Delta t V_{j,t,k} \ + \ \left(e^{-r_{k,t}\Delta t}\right)\left[\pi^{(u)}_{j,t,k} V_{j_u,t+1,k} \ + \ \pi^{(d)}_{j,t,k} V_{j_d,t+1,k}\right] \\
&= \delta_t \Delta t V_{j,t,k} \ + \ \left(e^{-r_{k,t}\Delta t}\right)\left[\pi^{(u)}_{j,t,k} u V_{j,t,k}\left(1-\delta_t\Delta t\right) \ + \ \pi^{(d)}_{j,t,k} d V_{j,t,k}\left(1-\delta_t\Delta t\right)\right] \\
&= V_{j,t,k}\left[\delta_t\Delta t \ + \ (1-\delta_t\Delta t)\left(e^{-r_{k,t}\Delta t}\right)\underbrace{\left(\pi^{(u)}_{j,t,k} u \ + \ \pi^{(d)}_{j,t,k} d\right)}_{= e^{r_{k,t}\Delta t} \text{ (No Arbitrage)}}\right] \\
&= V_{j,t,k}\left[\delta_t\Delta t \ + \ (1-\delta_t\Delta t)\left(e^{-r_{k,t}\Delta t}\right)(e^{r_{k,t}\Delta t})\right] \\
&= V_{j,t,k}\left[\delta_t\Delta t \ + \ 1 - \delta_t\Delta t\right] = V_{j,t,k}.
\end{aligned}$$

Part 2 of Proof

At maturity, T, M&M clearly holds ($E_{j,T,k} + D_{j,t,k} = V_{j,T,k}$):

$$\begin{aligned} E_{j,T,k} &= \max(0,\ V_{j,T} - (B_T + c_T)) \\ D_{j,T,k} &= \min(V_{j,T},\ B_T + c_T). \end{aligned}$$

Now we show that M&M holds at any arbitrary time $t \in \{1, ..., T-1\}$. Assuming $E_{j,t+1,k} + D_{j,t+1,k} = V_{j,t+1,k}$ (true for $t+1 = T$) and using induction:

$$E_{j,t,k} = \max\left(0,\ \delta_t \Delta t V_{j,t,k} - c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{E}_{t+1}\right],\ V_{j,t,k} - c_t - B_t - P_{j,t,k}\right)$$

$$D_{j,t,k} = \min\left(V_{j,t,k},\ c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right],\ c_t + B_t + P_{j,t,k}\right).$$

By hypothesis, $E_{j,t+1,k} + D_{j,t+1,k} = V_{j,t+1,k}$. So the continuation state value of date t equity =

$$\begin{aligned}
&\delta_t \Delta t V_{j,t,k} - c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{E}_{t+1}\right] \\
&= \underbrace{\left(\delta_t \Delta t V_{j,t,k} + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{V}_{t+1}\right]\right)}_{=V_{j,t,k} \text{ by Result 1}} - c_t - \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right] \\
&= V_{j,t,k} - c_t - \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right].
\end{aligned}$$

So,

$$\begin{aligned}
E_{j,t,k} &= \max\left(0,\ V_{j,t,k} - c_t - \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right],\ V_{j,t,k} - c_t - B_t - P_{j,t,k}\right) \\
&= V_{j,t,k} \ +\ \max\left(\underbrace{-V_{j,t,k}}_{-x_{j,t,k}},\ \underbrace{-c_t - \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right]}_{-y_{j,t,k}},\ \underbrace{-c_t - B_t - P_{j,t,k}}_{-z_{j,t,k}}\right).
\end{aligned}$$

Therefore:

$$\begin{aligned}
E_{j,t,k} + D_{j,t,k} = V_{j,t,k} &+ \max(-x_{j,t,k},\ -y_{j,t,k},\ -z_{j,t,k}) + \\
&+ \min(x_{j,t,k},\ y_{j,t,k},\ z_{j,t,k}) = V_{j,t,k}.
\end{aligned}$$

We can easily show M&M holds at $t = 0$ as well. Taking the appraised property value at origination S_0, we divide by $(1 - \delta_t \Delta t)$ to get V_0.

$$E_0 = (e^{-r_0 \Delta t}) \mathop{\mathbb{E}}_{0,0,0}[\tilde{E}_{t+1}]$$
$$D_0 = (e^{-r_0 \Delta t}) \mathop{\mathbb{E}}_{0,0,0}[\tilde{D}_{t+1}]$$
$$V_0 = \frac{S_0}{(1 - \delta_t \Delta t)} \Leftrightarrow S_0 = V_0 - \delta_t \Delta t V_0.$$

Using the continuation values of equity and debt for $t \in \{1, ..., T-1\}$ referenced above and removing the dividend $\delta_t \Delta t V_0$ and coupon c_t, we get:

$$E_0 = S_0 - (e^{-r_0 \Delta t}) \mathop{\mathbb{E}}_{0,0,0}[\tilde{D}_{t+1}]$$
$$E_0 + D_0 = S_0 - (e^{-r_0 \Delta t}) \mathop{\mathbb{E}}_{0,0,0}[\tilde{D}_{t+1}] + (e^{-r_0 \Delta t}) \mathop{\mathbb{E}}_{0,0,0}[\tilde{D}_{t+1}] = S_0.$$

Appendix D: Loss given default

We define LGD at each default node j, t, k as follows:

$$LGD_{j,t,k} = 1 - \frac{V_{j,t,k}}{c_t + B_t}.$$

For each loan in our sample, we obtain an expected LGD figure based on a Monte Carlo simulation run 10,000 times. To do this, we randomly determine property and interest rate movements by weighting these choices by their respective risk neutral probabilities. Upon reaching a default node, the simulation stops and records LGD for simulation number i as $LGD_i = LGD_{j,t,k}$. If no default occurs, $LGD_i = 0$. With probability $\lambda\Delta t$, a catastrophic property loss happens (the risk neutral "up" and "down" property probabilities sum to $1 - \lambda\Delta t$) and default occurs with $LGD_i = 1$. So expected LGD for each loan l is:

$$eLGD_l = \left(\Sigma_{i=1}^{10000} LGD_i\right) / 10000.$$

Appendix: Prepayment penalties

We use standard definitions for yield maintenance and defeasance, but modified to fit our term structure models.

The basic principle of defeasance is that the lender is losing a spread when the borrower refinances and requires the risk-free present value of that spread as a penalty. To mimic this spread, we use our term structure calculations to create a portfolio of risk-free assets (in our case, zero coupon bonds) with the same cash flows. The calculation for any interest rate state k, t is as follows:

$$Def_{k,t} = \sum_{i=1}^{T-t} \left(m_{t+i}\ ZCB_{k,t,t+i}\right) - B_t,$$

where m_{t+i} is the mortgage payment at date $t+i$, $ZCB_{k,t,t+i}$ is the value at date t of a zero coupon bond with maturity at date $t+i$, and B_t is the remaining mortgage balance.[26] $ZCB_{k,t,t+i}$ is calculated by creating a sub-tree M of all continuation

[26]Note that, for simplicity, we calculate defeasance and yield maintenance up to maturity T. In practice, there is some heterogeneity, with some lenders calculating the penalty up to the beginning of the open prepayment period instead. We do not observe the exact lender method in our data, though, and the differences between the beginning of the open prepayment period and maturity are usually very minor.

states starting at node k, t of the trinomial interest rate tree. The final column of the tree, which represents time $t+i$, has payoffs of 1 ($M_{k,t+i+1} = 1 \; \forall k$). We then determine the ZCB price by iterating backwards to the original node k, t so that the following recursive formula holds:

$$M_{k,t} = \left(e^{-r_{k,t}\Delta t}\right) \left[i_{k,t}^{(u)} M_{k_u,t+1} + i_{k,t}^{(m)} M_{k_m,t+1} + i_{k,t}^{(d)} M_{k_d,t+1}\right].$$

Yield maintenance is slightly different in that it involves replacing the missing spread with a U.S. Treasury security or other risk-free asset of the same remaining term as the mortgage. This is done using the calculated zero coupon bond rates as follows. First we calculate a "risk-free" par bond prevailing rate for the appropriate maturity:

$$rf_{k,t} = (1 - ZCB_{k,t,T}) \; / \; \left(\sum_{i=1}^{T-t} ZCB_{k,t,t+i}\right)$$

Then we calculate an annual "present value factor" f:

$$f_{k,t} = \left((1 - (1 + rf_{k,t}))^{-(T-t)/\Delta t}\right) / \; rf_{k,t}.$$

Finally, the yield maintenance penalty is calculated:

$$YM_{k,t} = (rm - rf_{k,t}) \, f_{k,t} \, B_t,$$

where rm is the mortgage rate.

Appendix F: Additional figures and tables

Table F1—Distribution of CRE Loans by Property Type

This table shows the absolute frequencies of commercial real estate mortgage loans in the Morningstar dataset across different collateral property types, with an emphasis on the distinctions between single and multi-property loan frequencies. The sample contains fixed-rate loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities.

	Single-Property Loans	Multi-Property Loans	Total
Hotel	4,808	194	5,002
Industrial	3,052	133	3,185
Mixed	0	174	174
Multi-family	18,972	491	19,463
Office	9,308	215	9,523
Other	5,903	331	6,234
Retail	16,084	292	16,376
Total	58,127	1,830	59,957

Table F2—Extreme Percentiles of Implied Volatility Estimates by Epoch

This table shows the lowest and highest three percentiles of implied volatility by time period (epoch). Epoch choice is explained in Section II.C. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities.

	P1	P2	P3	P97	P98	P99
2000–2004	0.06	0.08	0.09	0.36	0.39	0.44
2005–2007	0.08	0.09	0.10	0.32	0.34	0.39
2008–2015	0.06	0.08	0.09	0.32	0.34	0.37
2016–2020	0.10	0.12	0.13	0.36	0.38	0.40
Entire sample	0.07	0.08	0.09	0.34	0.36	0.40

Figure F1. Distribution of Implied Volatility by Epoch

This figure shows the distribution of calculated implied volatility by time period (epoch). Epoch choice is explained in Section II.C. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities.

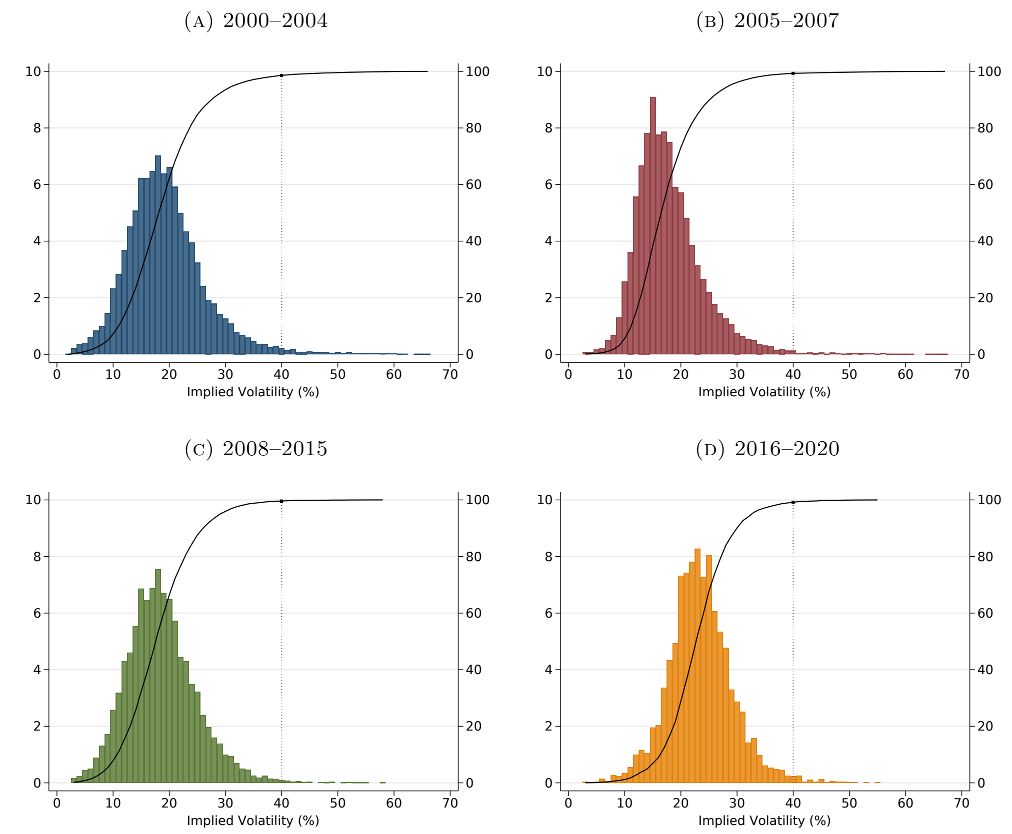
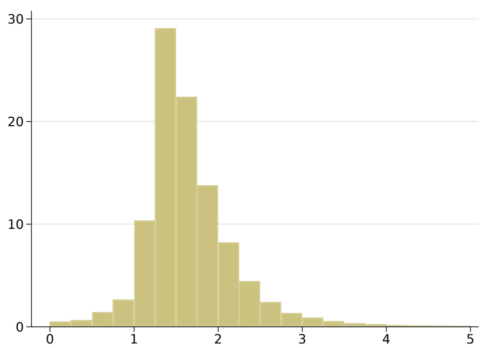


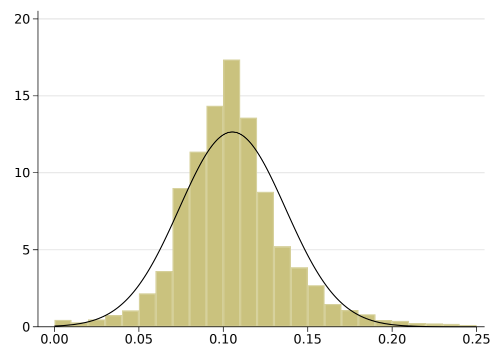
Figure F2. Distribution of Selected CRE Loan Characteristics

This figure shows the distribution of debt service coverage ratios, debt yield, loan-to-value ratios, and loan term lengths (months) in the Morningstar dataset of fixed-rate, single-property loans securitized in non-agency commercial mortgage-backed securities. The sample is later cut for further analysis by removing debt yields under 7% and debt service coverage ratios under 1.25 for reasons explained in section III.A.

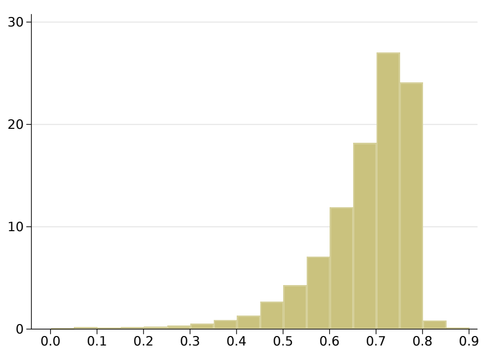
(a) DSCR



(b) Debt Yield



(c) Loan-to-Value Ratio



(d) Loan Term (Month)

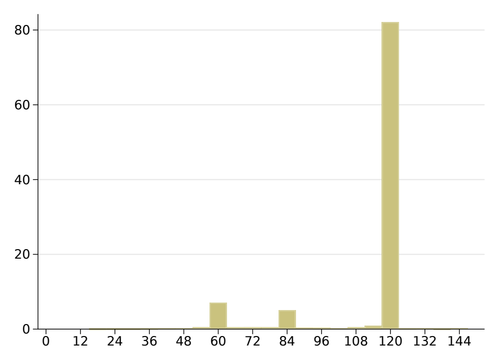


Figure F3. Association Between the LTVs and IVs of CMBS Loans Over Different Epochs

This figure shows the relationship between loan-level loan-to-value ratios and calculated implied volatility by time period (epoch). Epoch choice is explained in Section II.C. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The overlaid frontiers are estimated by fitting a quantile regression model for the 95$^{\text{th}}$ percentile of the loan-to-value ratios (LTVs) of commercial real estate loans that fall into a given integer bin of model-implied volatility and were originated in a given epoch. The samples contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities.

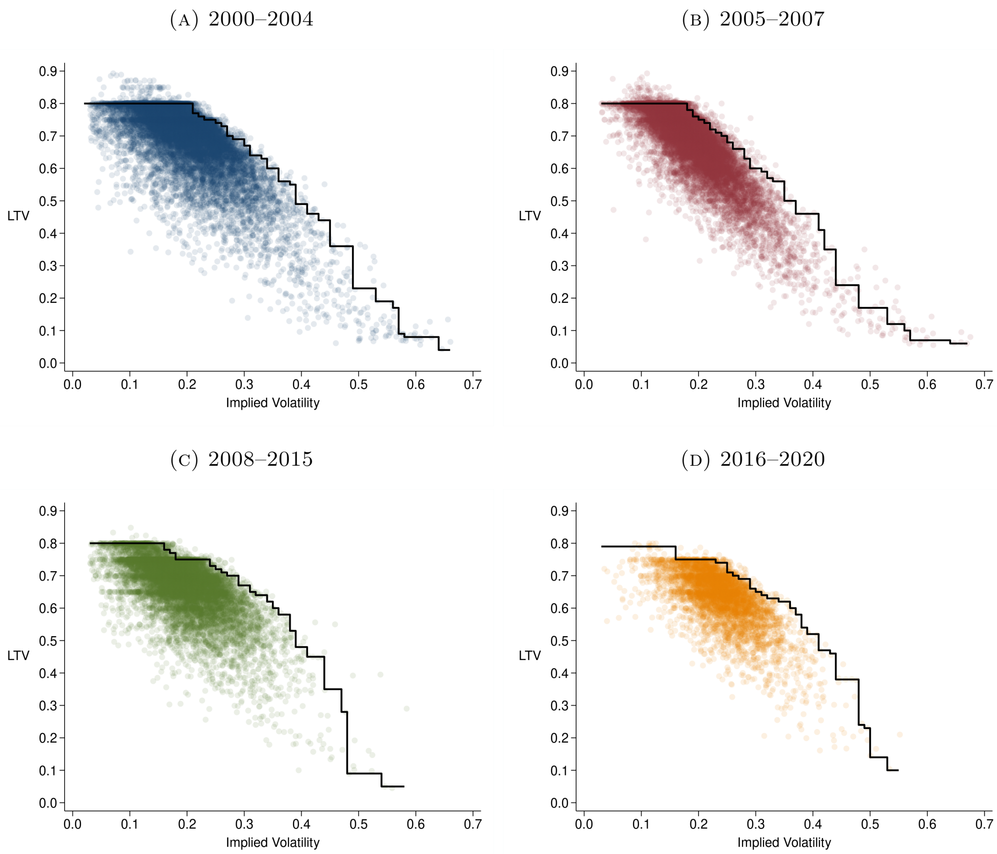


Table F3—Marginal Effects of Explanatory Variables on 100 × Log of Implied Volatility

This table shows the marginal effects of these explanatory variables on 100 × log of implied volatility. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The samples contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. Property size is measured alternatively in square feet and "units" by Morningstar. Rather than attempt a conversion, we utilize either of these variables when it is available. Certain variables are heterogeneously populated among property types, motivating us to include and exclude hotel, multi-family, and "other" property types in the various columns. GDP, income, and unemployment data are obtained from the Bureau of Economic Analysis (BEA). "Sector" refers to the "real estate industry" as defined by the BEA. The market size and vacancy rate variables are obtained from CBRE.

	(1)	(2)	(3)	(4)	(5)
100 × Log of State Real GDP (USD mm)	0.02	0.07	-0.00		
100 × Log of State Real GDP in Sector (USD mm)	0.05	-0.03	-0.01		
100 × Log of State Income per Capita (USD)	-0.20*	-0.11	-0.06		
State Unemployment Rate (%)	-1.36**	-1.19*	-1.20**		
Property Age (Years)	0.08***	0.08***	0.08***	0.05*	0.05**
100 × Log of Property Size (Sqft)	-0.04***	-0.04***	-0.04***	-0.04***	-0.03***
100 × Log of Property Size (Units)	-0.42***		-0.42***		-0.39***
Property Occupancy (%)		-0.10***		-0.16***	
Lead Tenant Share (%)		0.05***		0.05***	
100 × Log of Market Size (Sqft)			0.03***		0.01
Market Vacancy Rate (%)			-0.03		0.27***
Hotel Included	x		x		x
Multi-Family Included	x		x		x
Other Included	x	x		x	
Loan Originator	x	x	x	x	x
Property State	x	x	x		
Property Type × Time (Quarterly)	x	x	x	x	x
Property County × Time (Quarterly)				x	x
R^2	0.37	0.31	0.38	0.51	0.54
Number of Observations	41,362	19,626	26,433	13,185	21,209

Standard errors are double clustered by the property state and the quarter of origination.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table F4—Marginal Effects of Explanatory Variables on Implied Volatility (%)

This table shows the marginal effects of these explanatory variables on percentage implied volatility. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The samples contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. Property size is measured alternatively in square feet and "units" by Morningstar. Rather than attempt a conversion, we utilize either of these variables when it is available. Certain variables are heterogeneously populated among property types, motivating us to include and exclude hotel, multi-family, and "other" property types in the various columns. GDP, income, and unemployment data are obtained from the Bureau of Economic Analysis (BEA). "Sector" refers to the "real estate industry" as defined by the BEA. The market size and vacancy rate variables are obtained from CBRE.

	(1)	(2)	(3)	(4)	(5)
100 × Log of State Real GDP (USD mm)	0.004	0.013	-0.001		
100 × Log of State Real GDP in Sector (USD mm)	0.009	-0.006	-0.002		
100 × Log of State Income per Capita (USD)	-0.037*	-0.020	-0.011		
State Unemployment Rate (%)	-0.253**	-0.223*	-0.221**		
Property Age (Years)	0.014***	0.015***	0.014***	0.010*	0.010**
100 × Log of Property Size (Sqft)	-0.007***	-0.007***	-0.007***	-0.007***	-0.006***
100 × Log of Property Size (Units)	-0.077***		-0.077***		-0.071***
Property Occupancy (%)		-0.019***		-0.029***	
Lead Tenant Share (%)		0.010***		0.009***	
100 × Log of Market Size (Sqft)			0.005***		0.002
Market Vacancy Rate (%)			-0.005		0.050***
Hotel Included	x		x		x
Multi-Family Included	x		x		x
Other Included	x	x		x	
Loan Originator	x	x	x	x	x
Property State	x	x	x		
Property Type × Time (Quarterly)	x	x	x	x	x
Property County × Time (Quarterly)				x	x
Number of Observations	41,362	19,626	26,433	13,185	21,209

Standard errors are double clustered by the property state and the quarter of origination.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table F5—Marginal Effects of Explanatory Variables on IV (%)

This table shows the marginal effects of these explanatory variables on percentage implied volatility. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. This sample contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. This sample excludes hotel and multi-family properties.

	Pre-GFC	Post-GFC
Property Age (Years)	0.009*	0.012
100 × Log of Property Size (Sqft)	-0.008***	-0.004***
Property Occupancy (%)	-0.025***	-0.072***
Lead Tenant Share (%)	0.006**	0.027***
Number of Observations	10,573	2,612

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table F6—Marginal Effects of Explanatory Variables on IV (%)

This table shows the marginal effects of these explanatory variables on percentage implied volatility. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. This sample contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in non-agency commercial mortgage-backed securities. This sample excludes the "other" property type. The market vacancy rate is obtained from CBRE.

	Pre-GFC	Post-GFC
Property Age (Years)	0.010***	0.009*
100 × Log of Property Size (Sqft)	-0.006***	-0.006***
100 × Log of Property Size (Units)	-0.062***	-0.070***
100 × Log of Market Size (Sqft)	0.002	0.002
Market Vacancy Rate (%)	0.051*	0.050
Number of Observations	13,814	7,395

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$